Maintenance Planning as Complex System. Part II
(see Part I in Lecture 15)
Problem Statement

Assume that energy-producing company is going to set up a maintenance plan for power-producing equipment for a given planning period T_{max}, e.g., a month or a year. An array of m demands for maintenance work of power units is generated. The problem is to satisfy these demands. To do that we must include the maintenance work for all the demanded units into the plan, i.e., to schedule maintenance. A maintenance work of a power unit causes turning off of this unit, and, consequently, a fall of generating power in the system. Thus, it is impossible to satisfy all the demands because of problem constraints, which is basically the power reserve, i.e., the amount of power to be lost without turning off customers. This amount varies daily.

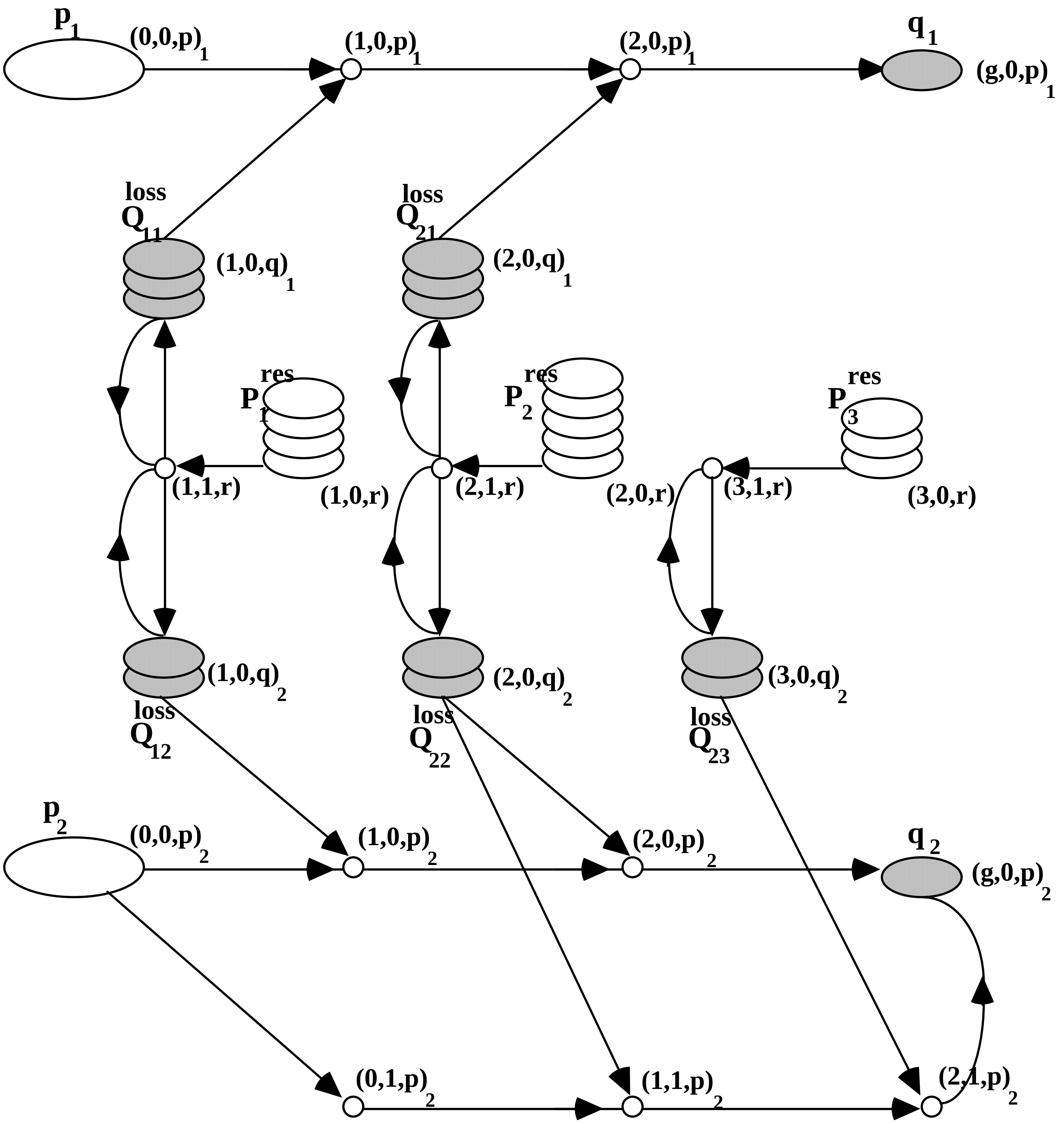
Each demand requests maintenance work for one power unit (j-th unit) and contains three attributes:

w_j, the demanded power of the unit;

h_j, the fall (loss) in the operating power of the energy-producing system because of maintenance of this unit (resources requirement); and

x_j^{max}, the required duration of maintenance.

For simplicity, we neglect the rest of the demand parameters. For the same reason we specify the only type of constraints function, $f(i)$ of power reserve for the energy-producing system, where i is the number of a day of the planning period. On the i-th day of the planning period the total fall (loss) in the operating power, because of the maintenance of some power units, can not be greater than the value $f(i)$. The values of all the parameters are positive integer numbers. The optimum criterion of the plan is the maximum total demanded power of the units being maintained.


p_1
$(0,0,p)_1$
$(1,0,p)_1$
$(2,0,p)_1$
q_1
$(g,0,p)_1$
loss
Q_{11}
$(1,0,q)_1$
loss
Q_{21}
$(2,0,q)_1$
res
P_1
$(1,0,r)$
$(1,1,r)$
res
P_2
$(2,0,r)$
$(2,1,r)$
res
P_3
$(3,1,r)$
$(3,0,r)$
$(1,0,q)_2$
loss
Q_{12}
$(2,0,q)_2$
loss
Q_{22}
$(3,0,q)_2$
loss
Q_{23}
p_2
$(0,0,p)_2$
$(1,0,p)_2$
$(2,0,p)_2$
q_2
$(g,0,p)_2$
$(0,1,p)_2$
$(1,1,p)_2$
$(2,1,p)_2$

A Class of Problems

A ***Complex System***

is the following eight-tuple

$< X, P, R_p, \{ON\}, v, S_i, S_t, TR>$

$X = \{x_i\}$ is a finite set of ***points***;

$P= \{p_i\}$ is a finite set of ***elements***; **$P= P_1 \cup P_2$, $P_1 \cap P_2=\emptyset$**;

$R_p(x, y)$ is a family of binary relations of ***reachability*** in X ($x \in X$, $y \in X$, $p \in P$); y is ***reachable*** from x for p;

ON(p)=x is a partial function of ***placement*** of elements P into X;

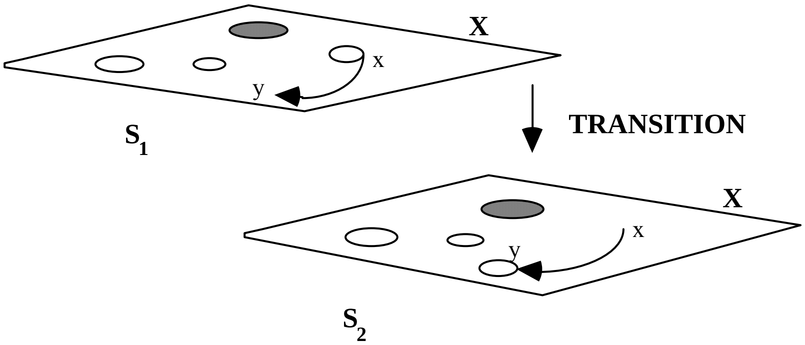
v> 0 is a real function, $v(p_i)$ are the ***values*** of elements;

S_i is a set of ***initial*** states of the system, a certain set of formulas $\{ON(p_i)=x_i\}$;

S_t is a set ***target*** states of the system (as S_i);

TR is a set of operators **TRANSITION(p, x, y)** for transition of the system from one state to another described as follows

precondition: $ON(p) = x \wedge R_p(x, y)$
delete: $ON(p) = x$, $ON(q) = y$
add: $ON(p) = y$



Formal representation of the Complex System for the maintenance problem is as follows:

$$\mathbf{X} = (\mathbf{Y} \cup \{\mathbf{g}\}) \times \mathbf{Y} \times (\mathbf{P}_{\mathrm{dem}} \cup \mathbf{Q}_{\mathrm{dem}} \cup \{\mathbf{r}\}),$$
$$\text{where } \mathbf{Y} = \{\mathbf{0}, \mathbf{1}, \ldots, \mathbf{T}_{\mathrm{max}}\},$$

P_{dem} is the set of power units included in the demands, $|P_{dem}|$ is the number of demands. A duplicate set Q_{dem} of the elements q_j is introduced, and one-to-one correspondence $q_j \leftrightarrow p_j$ is established between elements of $\mathbf{Q}_{\mathbf{dem}}$ and $\mathbf{P}_{\mathbf{dem}}$

$\mathbf{P} = \mathbf{P}_1 \cup \mathbf{P}_2$, $\mathbf{P}_1$ and $\mathbf{P}_2$ are not intersected and

$$\mathbf{P}_1 = \mathbf{P}_{\mathbf{dem}} \cup \mathbf{P}_{\mathbf{reserve}}, \quad \mathbf{P}_2 = \mathbf{Q}_{\mathrm{dem}} \cup \mathbf{Q}_{\mathrm{loss}},$$

$$\mathbf{P}_{\mathbf{reserve}} = \bigcup_{\mathbf{i=1}}^{\mathbf{Tmax}} \mathbf{P}_{\mathrm{i}}^{\mathrm{res}}, \quad \mathbf{Q}_{\mathbf{loss}} = \bigcup_{\mathbf{i=1}}^{\mathbf{Tmax}} \bigcup_{\mathbf{j=1}}^{|\mathbf{Qdem}|} \mathbf{Q}_{\mathrm{ij}}^{\mathrm{loss}}$$

To show the number of elements $|P_{reserve}|$ and $|Q_{loss}|$ we have to define $\mathbf{v_0}$. It is the quantum of power loss, the common factor of all values $f(i)$ of power reserve and all values h_j of power loss (for all demanded units); for example, $\mathbf{v_0}$=1 Megawatt. We can now determine $|P_{reserve}|$ and $|Q_{loss}|$, having given $|P_i^{res}|$ and $|Q_{ij}^{loss}|$. Thus,

$$|\mathbf{P}_{\mathbf{i}}^{\mathbf{res}}| = f(i)/\mathbf{v_0} \quad \text{and} \quad |\mathbf{Q}_{\mathbf{ij}}^{\mathbf{loss}}| = h_j/\mathbf{v_0}.$$

$R_p(x,y)$ can be defined explicitly by setting the values for all the triples of p, x, y:

If

$p \in P_{dem}$ **then**

$$R_p(x, y) = ((x = (0, y_2, p)) \wedge (y = (0, y_2+1, p))) \vee$$
$$((x = (y_1, y_2, p)) \wedge ((y = (y_1+1, y_2, p))) \vee$$
$$((x = (x_i^{max}, y_2, p)) \wedge (p = p_i) \wedge (y =(g,0, p)));$$

$p \in Q_{y_1 j} \subset Q_{loss}$ **then**

$$R_p(x, y) = ((x = ((y_1, 0,qj)) \wedge (y_1 > 0)) \wedge$$
$$(((y=(y_1-y_2,y_2,pj)) \wedge ((y_1-y_2)>0)) \vee (y=(y_1,1, r)));$$

$p \in P_{reserve}$ **then**

$$R_p(x, y) = (((x = (y_1, 0, r)) \wedge (y = (y_1, 1, r))) \vee$$
$$((x = (y_1, 1, r)) \wedge (y = (y_1, 0, q_j)) \wedge$$
$$(y_1 > 0) \wedge (q_j \wedge Q_{dem});$$

$p \in Q_{dem}$ **then**

$$R_p(x, y) = \text{F (false)}.$$

Note that here the reachability relation R_p is ***asymmetric***, i.e., there exist p, x, and y such that $R_p(x, y) \neq R_p(y, x)$.

To specify the partial function ON(p), it is sufficient to write down its values in the start state S_o:

If

$p \in P_{dem}$ then $ON(p) = (0, 0, p)$;

$p = q_j \in Q_{dem}$ then $ON(p) = (g, 0, p_j)$;

$p \in P_{y_1}^{res} \subset P_{reserve}$ then $ON(p) = (y_1, 0, r)$;

$p \in Q_{y_1 j}^{loss} \subset Q_{loss}$ then $ON(p) = (y_1, 0, q_j)$.

Function v(p) for target elements p = q_i is equal to the demanded power of separate power unit p_i; for the elements p_i, striving to reach targets, it is equal to the total power of all the demands, and for elements p of power reserve and loss v(p) equals to v_o, the quantum of the power loss.

$\mathbf{v_o}$ is the quantum of power loss, the common factor of all values *f(i)* of power reserve and all values h_j of power loss (for all demanded units); for example, $\mathbf{v_o}$ = 1 Megawatt.

If

$$p = q_i \in Q_{dem} \quad \text{then} \quad v(p) = w_i;$$

$$p \in P_{dem} \quad \text{then} \quad v(p) = \sum_{i=1}^{|Q_{dem}|} w_i;$$

$$p \in P_{reserve} \cup Q_{loss} \quad \text{then} \quad v(p) = v_o.$$

The operation of the system can easily be described using formulas for the **TRANSITION operator**.

S_i, the initial state, corresponds to the state of the energy-producing system in the "zero day" of planning period, while the target states

S_t, the target states, correspond to the state of the system with the maximum total demanded power of units being maintained. Thus, states from S_t can be described as states of the energy-producing system by the end of the planning period, in which the WFF ON(p_i) = (g, 0, p_i) are true for numbers i such that $\sum v(q_i)$ is maximum (q_i from Q_{dem}).

T, the set of transitions, consists of the “moves” of the elements along the network. Following are the meanings of some transitions (see Fig. 4):

— **TRANSITION(p_i, x, (g, 0, p_i))** with removal of ON(q_i) = (g, 0, p_i) means completion of the maintenance of the unit p_i.

— **TRANSITION(p_i, x, (1, y_2, p_i))** with addition of ON(p_i) = (1, y_2, p_i) means the unit p_i being taken out for the maintenance work on the day y_2.

— **TRANSITION(p_i, (0, y_2, p_i), (0, y_2+1, p_i))** with addition of ON(p_i) = (0, y_2+1, p_i) and removal of ON(p_i)=(0, y_2, p_i) means that on the day y_2+1 unit p_i has not yet been taken out for maintenance in the given plan variant.

Scheduling problem as Complex System

1. A STATEMENT OF THE PROBLEM

Here we consider a way of transformation of the different real-world system into the Complex System.

Assume that energy-producing company is going to set up a maintenance plan for power-producing equipment for a given planning period $T_{\max}$, e.g., month, year. There exists an array of m demands for maintenance work of power units. The problem is to satisfy these demands. To do that we must include the maintenance work for all the demanded units into the plan, i.e., to schedule maintenance. A maintenance work of a power unit causes turning off of this unit, and, consequently, a loss of generating power in the system. Thus, it is impossible to satisfy all the demands because of problem constraints, which is basically the power reserve, i.e., the amount of power to be lost without turning off customers. This amount varies daily.

Each demand requests maintenance work for one power unit (j-th unit) and contains three attributes: w_j, the demanded power of the unit; h_j, the loss in the operating power of the energy-producing system because of maintenance of this unit (resources requirement); and $x_j^{\max}$, required duration of maintenance. For simplicity, we neglect the rest of the demand parameters. For the same reason we specify the only one type of constraints the function $f(i)$ of power reserve for the energy-producing system, where i is the number of a day of the planning period. On the i-th day of the planning period the total loss in the operating power, because of the maintenance of some power units, can not be greater then the value $f(i)$. The values of all the parameters are positive integer numbers.

The optimum criterion of the plan is the maximum total demanded power of the units being maintained.

In terms of the Complex System, this problem might be represented as a twin-set of *elements* and *points*, as depicted in the Figure. Here *points* form a network which is used by *elements* as a "railroad" to reach certain nodes. There are two classes of *elements*. The first one includes power units, depicted as white discs p_1, p_2, striving to reach nodes (g, 0, p_1) and (g, 0, p_2) and thereby gain opposite elements q_1, q_2 (i.e., the ones to be maintained). The other elements of the first class are depicted as pyramids of white disks P_i^{res}: each pyramid represents a daily stock of resources, the power reserve for the energy-producing system. The pyramids of opposite black discs Q_{ij}^{loss} represent requirements of resources, the daily loss in the operating power because of the maintenance of the units p_1 and p_2. The black discs control the nodes of paths for discs p_1, p_2 and able to gain any of them, i. e., maintenance can not take place without provision of resources. It means we are forced to spend white discs of pyramids P_i^{res}

exchanging them in the nodes (i, 1, r) with the black discs of Q_{ij}^{loss}. These actions can "clear away" the paths for power units p_1 and p_2.

2. A FORMAL REPRESENTATION

Formal representation of the Complex System for the maintenance problem is as follows:

$\mathbf{X} = (Y \cup \{g\}) \times Y \times (P_{dem} \cup Q_{dem} \cup \{r\})$, where $Y=\{0,1,...,T_{max}\}$,

P_{dem} is the set of power units included in the demands, $|P_{dem}|$ is the number of demands. It is introduced a duplicate set Q_{dem} of the elements q_j, and one-to-one correspondence $q_j \longleftrightarrow p_j$ is established between elements of Q_{dem} and P_{dem}

$\mathbf{P}=P_1 \cup P_2$, P_1 and P_2 are not intersected and

$$P_1= P_{dem} \cup P_{reserve},\quad P_2=Q_{dem} \cup Q_{loss},\quad P_{reserve}= \bigcup_{i=1}^{Tmax} P_i^{res},\quad Q_{loss}=\bigcup_{i=1}^{Tmax} \bigcup_{j=1}^{|Qdem|} Q_{ij}^{loss}$$

To show the number of elements $|P_{reserve}|$ and $|Q_{reserve}|$ we have to define $\mathbf{v_o}$. It is the quantum of power loss, the common factor of all values $f(i)$ of power reserve and all values h_j of power loss (for all demanded units); for example, $\mathbf{v_o}=1$ Megawatt. We can now determine $|P_{reserve}|$ and $|Q_{loss}|$, having given $|P_i^{res}|$ and $|Q_{ij}^{loss}|$. Thus, $|P_i^{res}| = f(i)/\mathbf{v_o}$ and $|Q_{ij}^{loss}| = h_j/\mathbf{v_o}$.

The relation of reachability $R_p(x,y)$ can be given explicitly by setting the values for all the triples of p, x, y:

$$\mathbf{R_p(x,y)} = \begin{cases} ((x=(0, y_2, p)) \wedge (y-(0, y_2+1, p))) \vee ((x=(y_1, y_2, p)) \wedge \\ ((y=(y_1+1, y_2, p))) \text{ v } ((x=(x_i^{max}, y_2, p)) \wedge (p=p_i) \wedge \\ (y=(g, 0, p))), & \text{if p from } P_{dem}; \\ \\ ((x=((y_1, 0, q_j)) \wedge (y_1>0)) \wedge (((y=(y_1-y_2, y_2, p_j)) \wedge \\ ((y_1-y_2)>0)) \vee (y=(y_1, 1, r))), & \text{if p from } Qy_{1}j^{loss}, \\ & \text{a subset of } Q_{loss} \\ (((x=(y_1, 0, r)) \wedge (y=(y_1, 1, r))) \vee ((x=(y_1, 1, r)) \wedge \\ (y=(y_1, 0, q_j)) \wedge (y_1>0) \wedge (q_j \in Q_{dem}), & \text{if p from } P_{reserve}; \\ \\ F \text{ (false)}, & \text{if p from } Q_{dem}. \end{cases}$$

Note that here the reachability relation R_p is *asymmetric*, i.e., there exist p, x, and y such that $R_p(x, y) \neq R_p(y, x)$. To specify the partial function ON(p), it is sufficient to write out its values in the initial state S_o:

$$\textbf{ON(p)=} \quad \begin{matrix} (0, 0, p), & \text{if } p \text{ from } P_{dem}; \\ (g, 0, p_j), & \text{if } p=q_j \text{ from } Q_{dem}; \\ (y_1, 0, r) & \text{if } p \text{ from } P^{res}_{y_1}\text{, a subset of } P_{reserve}; \\ (y_1, 0, q_j) & \text{if } p \text{ from } Q^{loss}_{y_1 j}\text{, a subset of } Q_{loss}. \end{matrix}$$

Function v(p) for target elements $p=q_i$ is equal to the demanded power of separate power unit p_i; for the elements p_i, striving to reach targets, it is equal to the total power of all the demands, and for elements p of power reserve and loss v(p) equals to v_o, the quantum of power loss (see above).

$$\textbf{v(p)=} \begin{cases} w_i, & \text{if } p=q_i \text{ from } Q_{dem}; \\ \sum_{i=1}^{|Q_{dem}|} w_i, & \text{if } p \text{ from } P_{dem}; \\ \mathbf{v_o} & \text{if } p \text{ from } P_{reserve} \cup P_{loss}. \end{cases}$$

The functioning of the system can easily be described using formulas for the TRANSITION operator.

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S_t, the target states, correspond to the state of the system with the maximum total demanded power of units being maintained. Thus, states from S_t can be described as states of the energy-producing system by the end of the planning period, in which the WFF $ON(p_i)=(g, 0, p_i)$ are true for numbers i such that $\sum v(q_i)$ is maximum (q_i from Q_{dem}).

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Following are the meanings of some transitions (see Fig. 4):

— TRANSITION(p_i, x, (g, 0, p_i)) with removal of the WFF $ON(q_i)=(g, 0, p_i)$ means completion of the maintenance of the unit p_i.

— TRANSITION(p_i, x, (1, y_2, p_i)) with addition of the WFF $ON(p_i)=(1, y_2, p_i)$ means the unit p_i being taken out for the maintenance work on the day y_2.

— TRANSITION(p_i, (0, y_2, p_i), (0, y_2+1, p_i)) with addition of the WFF $ON(p_i) = (0, y_2+1, p_i)$ and removal of $ON(p_i)=(0, y_2, p_i)$ means that on the day y_2+1 unit p_i

has not yet been taken out for maintenance in the given plan variant.

3. DETAILS OF MAINTENANCE PROBLEM

To clarify this problem let us return to the example depicted in Figure and consider it in details. This is the maintenance planning problem for two units over a period of three days:

$$w_1=5,\ w_2=2;\quad h_1=3,\ h_2=2;\quad x_1^{\max}=x_2^{\max}=2;\quad T_{\max}=3;\quad f(1)=4;\ f(2)=5;\ f(3)=3.$$

(A reader should not be confused by the simplicity of the example shown in the Figure. It is cited here only for clarification of our approach. For the practical applications there were considered hundreds and even thousands of power units, and different kinds of resources including those which required some time to be delivered to the places of maintenance.)

From Figure it is seen that, for setting up the maintenance plan, the elements p_i have to go from the points $(0, 0, p_i)$ to the points $(g, 0, p_i)$. In particular, for element p_2 to get through to the point $(g, 0, p_2)$ along any of the paths

$$(0, 0, p_2) \longrightarrow (1, 0, p_2) \longrightarrow (2, 0, p_2) \longrightarrow (g, 0, p_2)$$

or

$$(0, 0, p_2) \longrightarrow (0, 1, p_2) \longrightarrow (1, 1, p_2) \longrightarrow (g, 0, p_2),$$

it is necessary to do away with the elements of the set (pyramid) Q_{12}^{loss} at the point $(1, 0, q_2)$, as well as the elements of the pyramids Q_{22}^{loss}, Q_{23}^{loss} at the points $(2, 0, q_2)$, $(3, 0, q_2)$. The elements of these pyramids control the points of the path of the element p_2 to the target. Obviously, pyramids of elements from Q_{loss} correspond to loss of power in the energy-producing system during the time of maintenance of power units.

For liquidation of the elements from Q_{loss} we have three sets (pyramids of discs) P_1^{res}, P_2^{res}, P_3^{res} at the points $(1, 0, r)$, $(2, 0, r)$ and $(3, 0, r)$ corresponding to the power reserves in the system during each particular day. It is necessary to carry out a transition, i.e., to move an element from P_1^{res} to the point $(1, 1, r)$, then move an element from Q_{12}^{loss} to the same point, i. e., to perform a "capture", then move the next element from P_1^{res}, and so forth.

In the given example pyramids are placed at one-step distance from the points of exchange. It means the instantaneous availability of resources in the given problem. For complex real-world problems pyramids of resources have to be placed at several steps from the points of exchange which means that resources delivery should start in advance, in several time intervals.

Returning to our example, if at the point $(1, 1, r)$ it is possible to exchange all the elements from Q_{loss}, then the point $(1, 0, p_2)$ becomes traversable freely for the element p_2. If this, however, is not possible (as is in fact shown in Figure), owing to the fact that three elements of the pyramid P_1^{res} were spent on removing the control from the point $(1, 0, p_1)$, i.e., on liquidating Q_{11}^{loss}, and if the remaining single element is not sufficient for destroying the two elements of Q_{12}^{loss}, the element p_2 is forced to move to the point (0, 1, p_2). Thus, on the first day of the planning period, only one of the power units (p_1, for example) can be taken out for maintenance because of insufficiency of power reserve. The second unit p_2 will be taken out on the second day (displacement $(0, 1, p_2) \longrightarrow (1, 1, p_2)$). Different versions of the maintenance plan are matched by different variants of movement of elements from P along points from X.