Geometry of Zones

-----OLD DEFINITIONS----

A ***trajectory connection***
of the trajectories t_1 and t_2 is the relation $C(t_1,t_2)$. It holds, if the *ending link* of the trajectory t_1 coincides with an *intermediate link* of the trajectory t_2;

On the set **A** of trajectories it is defined:

$\mathbf{C_A^{\mathit{k}}(t_1, t_2)}$, a ***k-th degree of connection*** and

$\mathbf{C_A^{+}(t_1,t_2)}$, a ***transitive closure.***

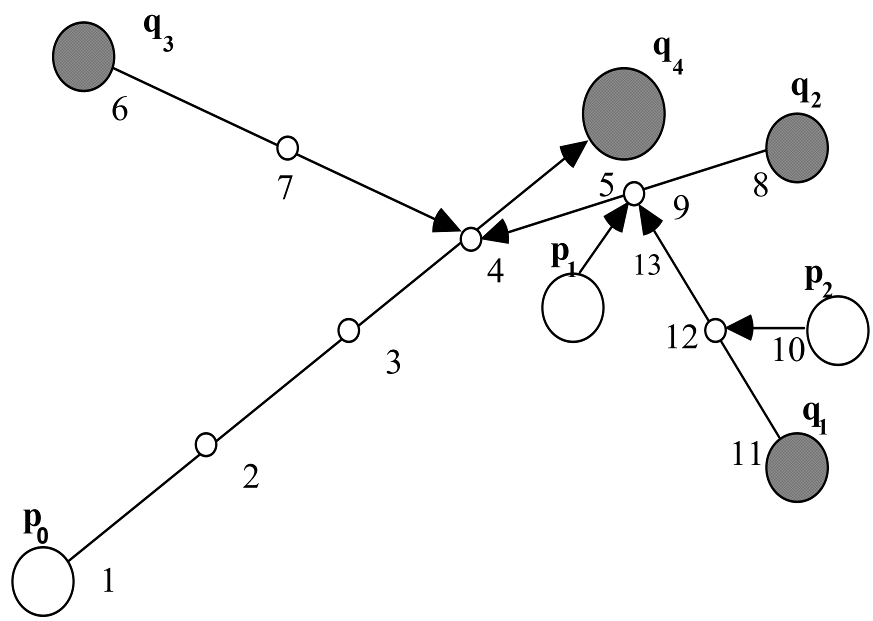
A ***trajectory network*** **W**
relative to trajectory t_o is a finite set of trajectories $t_o, t_1,\ldots, t_k$ from the language $L_t^H(S)$: for every trajectory t_i from W (i=1, 2,…, k) the relation $C_W^{+}(t_i, t_o)$ holds.

A ***family of trajectory network languages*** $\mathbf{L_C(S)}$
in a state S of the Complex System is the family of languages that contains strings of the form

$$\boldsymbol{t}(t_1,param)\boldsymbol{t}(t_2,param)\ldots\boldsymbol{t}(t_m,param),$$

where *param* in parentheses substitute for the other parameters of a particular language. All the symbols $t_1,t_2,\ldots,t_m$ correspond to trajectories which form a trajectory network **W** relative to t_1.

Network language interpretation.



Language of Zones

Definition
A language $\mathbf{L_Z(S)}$ generated by the grammar $\mathbf{G_Z}$ in a state S of a Complex System is called the ***Language of Zones***.

Grammar of Zones G_Z

L	Q	Kernel, π_k ($\forall z \in X$)	π_n ($\forall z \in X$)	F_T	F_F
1	Q_1	$S(u, v, w) \to A(u, v, w)$		*two*	$\varnothing$
2_i	Q_2	$A(u, v, w) \to t(h_i^o(u), l_o+1)$ $A((0, 0, 0), g(h_i^o(u),w), zero)$	$TIME(z) = DIST(z, h_i^o(u))$	3	$\varnothing$
3	Q_3	$A(u, v, w) \to A(f(u, v), v, w)$	$NEXTTIME(z) = init(u, NEXTTIME(z))$	*four*	5
4_j	Q_4	$A(u, v, w) \to t(h_j(u), TIME(y)))$ $A(u, v, g(h_j(u), w))$	$NEXTTIME(z) = ALPHA(z, h_j(u), TIME(y) - l+1)$	3	3
5	Q_5	$A(u, v, w) \to A((0, 0, 0), w, zero)$	$TIME(z) = NEXTTIME(z)$	3	6
6	Q_6	$A(u, v, w) \to \varepsilon$		$\varnothing$	$\varnothing$

$V_T = \{t\}$, $V_N = \{S, A\}$,

V_{PR}

Pred $= \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$

$Q_1(u) = (ON(p_o) = x) \wedge (MAP_{x,p_o}(y) \le l \le l_o) \wedge$
$(\exists q\ ((ON(q) = y) \wedge (OPPOSE(p_o, q))))$

$Q_2(u) = T$

$Q_3(u) = (x \ne n) \vee (y \ne n)$

$Q_4(u) = (\exists p\ ((ON(p) = x) \wedge (l > 0) \wedge (x \ne x_o) \wedge (x \ne y_o)) \wedge$
$((\neg OPPOSE(p_o, p) \wedge (MAP_{x,p}(y) = 1)) \vee$
$(OPPOSE(p_o, p) \wedge (MAP_{x,p}(y) \le l)))$

$Q_5(w) = (w \ne zero)$

$Q_6 = T$

Var $= \{x, y, l, \tau, \theta, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n\}$; for the sake of brevity:
$u = (x, y, l)$, $v = (v_1, v_2, ..., v_n)$, $w = (w_1, w_2, ..., w_n)$, $zero = (0, 0, ..., 0)$

Con $= \{x_o, y_o, l_o, p_o\}$; *Func* $= Fcon \cup Fvar$;

Fcon $= \{f_x, f_y, f_l, g_1, g_2, ..., g_n, h_1, h_2, ..., h_M,$
$h_1^o, h_2^o, ..., h_M^o, DIST, init, ALPHA\}$, $f = (f_x, f_y, f_l)$, $g = (g_{x1}, g_{x2}, ..., g_{xn})$,
$M = |\mathbf{L}_t^{l_o}(S)|$ is the number of trajectories in $\mathbf{L}_t^{l_o}(S)$

Fvar $= \{x_o, y_o, l_o, p_o, TIME, NEXTTIME\}$

$E = \mathbf{Z}_+ \cup X \cup P \cup \mathbf{L}_t^{l_o}(S)$ is the subject domain;

Parm: $S \varnothing Var$, $A \to \{u, v, w\}$, $t \to \{p, \tau, \theta\}$;

$L = \{1,3,5,6\} \cup two \cup four$, $two = \{2_1, 2_2, ..., 2_M\}$, $four = \{4_1, 4_2, ..., 4_M\}$

Definition of functions of the Grammar of Zones G_Z

D(*init*) = $X \times X \times \mathbf{Z}_+ \times \mathbf{Z}_+$

$$\boldsymbol{init}\ (u, r) = \begin{cases} 2n, & \text{if } u = (0,0,0), \\ r, & \text{if } u \neq (0,0,0). \end{cases}$$

D(***f***) = $(X \times X \times \mathbf{Z}_+ \cup \{0, 0, 0\}) \cup \mathbf{Z}_+^n$

$$f(u,\ v) = \begin{cases} (x+1, y, l), & \text{if } ((x \neq n) \wedge (l > 0)) \vee ((y = n) \wedge (l \leq 0)) \\ (1,\ y+1,\ TIME(y+1) \times v_{y+1}), & \text{if } (x = n) \vee ((l \leq 0) \wedge (y \neq n)). \end{cases}$$

D(*DIST*) = $X \times P \times \mathbf{L_t}^{l_o}(S)$.

Let $t_o \in L_t^{l_o}(S)$, $t_o = \boldsymbol{a}(z_o)\boldsymbol{a}(z_1)...\boldsymbol{a}(z_m)$, $t_o \in t_{p_o}(z_o, z_m, m)$;

If $((z_m = y_o) \wedge (p = p_o) \wedge (\exists\, k\ (1 \leq k \leq m) \wedge (x = z_k))) \vee$

$(((z_m \neq y_o) \vee (p \neq p_o)) \wedge (\exists\, k\ (1 \leq k \leq m - 1) \wedge (x = z_k)))$

then $DIST(x, p_o, t_o) = k+1$

else $DIST(x, p_o, t_o) = 2n$

D(*ALPHA*) = $X \times P \times \mathbf{L_t}^{l_o}(S) \times \mathbf{Z}_+$

$$ALPHA\ (x, p_o, t_o, k) = \begin{cases} max\ (NEXTTIME\ (x), k), & \text{if } (DIST\ (x, p_o, t_o) \neq 2n) \\ & \wedge\ (NEXTTIME\ (x) \neq 2n); \\ k, & \text{if } DIST\ (x, p_o, t_o) \neq 2n) \\ & \wedge\ (NEXTTIME\ (x) = 2n); \\ NEXTTIME\ (x), & \text{if } DIST\ (x, p_o, t_o) = 2n). \end{cases}$$

D(g_r) = $P \times \mathbf{L_t}^{l_o}(S) \times \mathbf{Z}_+^n$, $r \in X$.

$$g_{\mathbf{r}}(p_o, t_o, w) = \begin{cases} 1, & \text{if } DIST(r, p_o, t_o) < 2n, \\ w_r, & \text{if } DIST(r, p_o, t_o) = 2n. \end{cases}$$

D (h_i^o) = $X \times X \times \mathbf{Z}_+$; Let $TRACKS_{p_o} = \{p_o\} \times (\bigcup_{1 \leq k \leq l} L[\mathbf{G_t}^{(2)}(x, y, k, p_o)]$

If $TRACKS_{p_o} = \varnothing$

then $h_i^o(u) = \varepsilon$

else $TRACKS_{p_o} = \{(p_o, t_1), (p_o, t_2), \ldots, (p_o, t_b)\}, (b \leq M)$ **and**

$$h_i^o(u) = \begin{cases} (p_o, t_i), & \textbf{if } i \leq b, \\ (p_o, t_b), & \textbf{if } i > b. \end{cases}$$

D(h_i) = $X \times X \times \mathbf{Z}_+$; Let $TRACKS_p = \{p\} \times (\bigcup_{1 \leq k \leq l} L[\mathbf{G_t}^{(2)}(x, y, k, p)]$

If TRACKS$_p = \varnothing$

then $h_i(u) = \varepsilon$

else TRACKS $_p = \{(p_1, t_1), (p_1, t_2), \ldots, (p_m, t_m)\}$, $(m \le M)$ **and**

$$h_i(u) = \begin{cases} (p_i, t_i), & \textbf{if } i \le m, \\ (p_m, t_m), & \textbf{if } i > m. \end{cases}$$

-

Trajectories t_i should not be embedded (as sub-trajectories) in the trajectories of the same negation.

At the beginning of generation:

$u = (x_0, y_0, l_0)$, $w = zero$, $v = zero$, $x_0 \in X$, $y_0 \in X$, $l_0 \in \mathbf{Z}_+$, $p_0 \in P$,

TIME (z) = 2n, *NEXTTIME* (z) = 2n for all z from X.

To study this language formally we need preliminary definitions.

Definition 1.
An ***alphabet*** **A(Z)** ***of the string*** **Z** of the parameter language **L** is the set symbols of this language with given parameter values, where each of the symbols with parameters is included at least once in a string **Z**, and ***e*** (empty symbol).

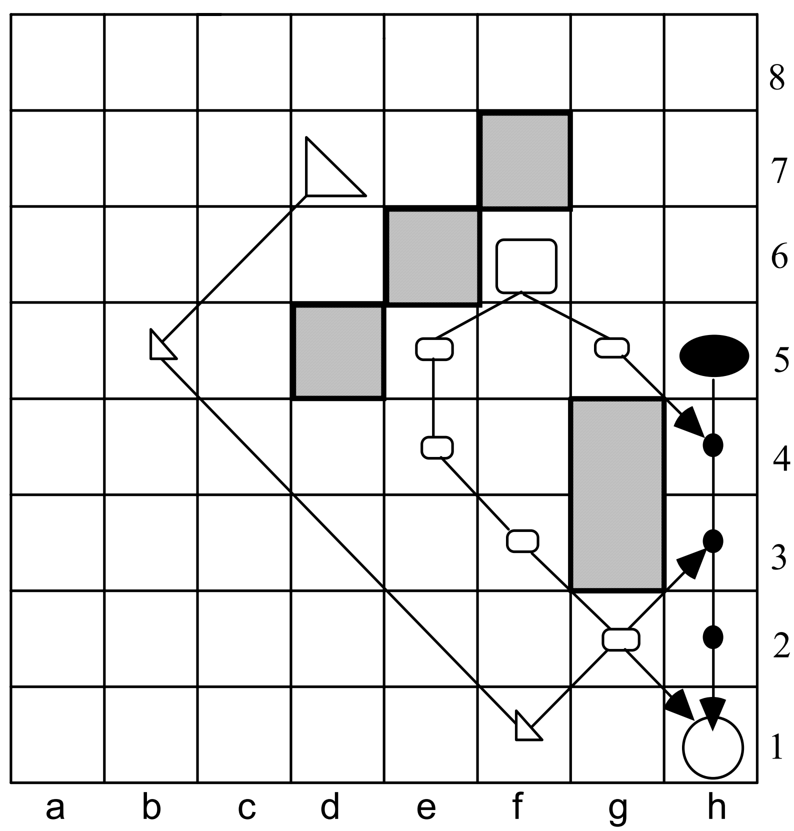
Definition 2.
A ***trajectory alphabet*** **TA(Z)** of the Zone **Z** is the set of trajectories from $\mathbf{L_t^H(S)}$ that correspond to the actual parameter values of the alphabet **A(Z)**.

Example of Zone

$\boldsymbol{t}$(BOMBER, t_B,5)$\boldsymbol{t}$(FIGHTER, t_F,5)$\boldsymbol{t}$(MISSILE, t_M,5)$\boldsymbol{t}$(MISSILE, $t_M{}^1$, 3) $\boldsymbol{t}$(FIGHTER, $t_F{}^1$, 2),

where

t_B=$\boldsymbol{a}$(h5)$\boldsymbol{a}$(h4)$\boldsymbol{a}$(h3)$\boldsymbol{a}$(h2)$\boldsymbol{a}$(h1),
t_F=$\boldsymbol{a}$(f6)$\boldsymbol{a}$(e5)$\boldsymbol{a}$(e4)$\boldsymbol{a}$(f3)$\boldsymbol{a}$(g2)$\boldsymbol{a}$(h1),
t_M=$\boldsymbol{a}$(d7)$\boldsymbol{a}$(b5)$\boldsymbol{a}$(f1)$\boldsymbol{a}$(g2)$\boldsymbol{a}$(h1),
$t_M{}^1$=$\boldsymbol{a}$(d7)$\boldsymbol{a}$(b5)$\boldsymbol{a}$(f1)$\boldsymbol{a}$(h3),
$t_F{}^1$=$\boldsymbol{a}$(f6)$\boldsymbol{a}$(g5)$\boldsymbol{a}$(h4)



Theorem
For any string Z from $L_Z(S)$, trajectories from TA(Z) form a trajectory network, i.e., $L_Z(S) \in L_C(S)$.

Proof
Let us consider a string $\mathbf{Z} = \boldsymbol{t}(\mathbf{p_o}, \mathbf{t_o}, \boldsymbol{\tau_o}) \ldots \boldsymbol{t}(\mathbf{p_k}, \mathbf{t_k}, \boldsymbol{\tau_k})$.

Grammar of Zones G_Z

Q		Kernel, π_k ($\forall z \in X$)	π_n ($\forall z \in X$)	F_T	F_F
1	Q_1	$S(u, v, w) \rightarrow A(u, v, w)$		*two*	$\varnothing$
2_i	Q_2	$A(u, v, w) \rightarrow t(h_i^o(u), l_o+1)$ $A((0, 0, 0), g(h_i^o(u),w), zero)$	$TIME(z) = DIST(z,h_i^o(u))$	3	$\varnothing$
3	Q_3	$A(u, v, w) \rightarrow A(f(u, v), v, w)$	$NEXTTIME(z) = init(u, NEXTTIME(z))$	*four*	5
4_j	Q_4	$A(u, v, w) \rightarrow t(h_j(u), TIME(y)))$ $A(u, v, g(h_j(u), w))$	$NEXTTIME(z) = ALPHA(z, h_j(u), TIME(y) - l+1)$	3	3
5	Q_5	$A(u, v, w) \rightarrow A((0, 0, 0), w, zero)$	$TIME(z) = NEXTTIME(z)$	3	6
6	Q_6	$A(u, v, w) \rightarrow \varepsilon$		$\varnothing$	$\varnothing$

$V_T = \{t\}$, $V_N = \{S, A\}$,

V_{PR}

Pred $= \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$

$Q_1(u) = (ON(p_o) = x) \wedge (MAP_{x,p_o}(y) \leq l \leq l_o) \wedge$
$(\exists q\ ((ON(q) = y) \wedge (OPPOSE(p_o, q))))$

$Q_2(u) = T$

Obviously under the condition that the predicate Q_1 is true, the symbol $\boldsymbol{t}(\mathbf{p_o}, \mathbf{t_o}, \boldsymbol{\tau_o})$ is attached to the string by applying the productions **1** and $\mathbf{2_i}$.

The following proof is by induction.

We assume that all the trajectories **TA(Z_m)** of the substring

$$Z_m = t(p_o, t_o, \tau_o) \ldots t(p_m, t_m, \tau_m)$$

form a trajectory network. Symbol $t(p_{m+1}, t_{m+1}, \tau_{m+1})$ can be attached to a string only after applying the production **4_j.**

L	Q	Kernel, π_k $(\forall z \in X)$	π_n $(\forall z \in X)$	F_T	F_F
1	Q_1	$S(u, v, w) \to A(u, v, w)$		*two*	$\varnothing$
2_i	Q_2	$A(u, v, w) \to t(h_i^o(u), l_o+1)$ $A((0, 0, 0), g(h_i^o(u), w), zero)$	$TIME(z) = DIST(z, h_i^o(u))$	3	$\varnothing$
3	Q_3	$A(u, v, w) \to A(f(u, v), v, w)$	$NEXTTIME(z) = init(u, NEXTTIME(z))$	*four*	5
4_j	Q_4	$A(u, v, w) \to t(h_j(u), TIME(y)))$ $A(u, v, g(h_j(u), w))$	$NEXTTIME(z) = ALPHA(z, h_j(u), TIME(y) - l+1)$	3	3
5	Q_5	$A(u, v, w) \to A((0, 0, 0), w, zero)$	$TIME(z) = NEXTTIME(z)$	3	6
6	Q_6	$A(u, v, w) \to \varepsilon$		$\varnothing$	$\varnothing$

$V_T = \{t\}$, $V_N = \{S, A\}$,

V_{PR}

$Pred = \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$

$Q_1(u) = (ON(p_o) = x) \wedge (MAP_{x,p_o}(y) \le l \le l_o) \wedge$
$(\exists q\, ((ON(q) = y) \wedge (OPPOSE(p_o, q))))$

$Q_2(u) = T$

$Q_3(u) = (x \ne n) \vee (y \ne n)$

$Q_4(u) = (\exists p\, ((ON(p) = x) \wedge (l > 0) \wedge (x \ne x_o) \wedge (x \ne y_o)) \wedge$
$((\neg OPPOSE(p_o, p) \wedge (MAP_{x,p}(y) = 1)) \vee$
$(OPPOSE(p_o, p) \wedge (MAP_{x,p}(y) \le l)))$

$Q_5(w) = (w \ne zero)$ $\qquad Q_6 = T$

Among the parameters of the trajectory $t_{m+1} \in t_p(x, y, l)$ we are interested in the value of **y**, the parameter value of the **last symbol** of the trajectory. One can pass to the production with the label **4_j** only after a successful application of a production with the label **3**, i.e., in F_T case. Here the $f(u, v)$ function changes the value of the parameter $u = (x, y, l)$.

L	Q	Kernel, π_k ($\forall z \in X$)	π_n ($\forall z \in X$)	F_T	F_F
1	Q_1	$S(u, v, w) \to A(u, v, w)$		*two*	$\varnothing$
2_i	Q_2	$A(u, v, w) \to t(h_i^o(u), l_o+1)$ $A((0, 0, 0), g(h_i^o(u),w), zero)$	$TIME(z) = DIST(z, h_i^o(u))$	3	$\varnothing$
3	Q_3	$A(u, v, w) \to A(f(u, v), v, w)$	$NEXTTIME(z) = init(u, NEXTTIME(z))$	*four*	5
4_j	Q_4	$A(u, v, w) \to t(h_j(u), TIME(y)))$ $A(u, v, g(h_j(u), w))$	$NEXTTIME(z) = ALPHA(z, h_j(u), TIME(y) - l+1)$	3	3
5	Q_5	$A(u, v, w) \to A((0, 0, 0), w, zero)$	$TIME(z) = NEXTTIME(z)$	3	6
6	Q_6	$A(u, v, w) \to \varepsilon$		$\varnothing$	$\varnothing$

$V_T = \{t\}$, $V_N = \{S, A\}$,

V_{PR}

$Pred = \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$

$Q_1(u) = (ON(p_o) = x) \wedge (MAP_{x,p_o}(y) \le l \le l_o) \wedge$
$(\exists q\, ((ON(q) = y) \wedge (OPPOSE(p_o, q))))$

$Q_2(u) = T$

$Q_3(u) = (x \ne n) \vee (y \ne n)$

$Q_4(u) = (\exists p\, ((ON(p) = x) \wedge (l > 0) \wedge (x \ne x_o) \wedge (x \ne y_o)) \wedge$
$((\neg OPPOSE(p_o, p) \wedge (MAP_{x,p}(y) = 1)) \vee$
$(OPPOSE(p_o, p) \wedge (MAP_{x,p}(y) \le l)))$

$Q_5(w) = (w \ne zero)$

$Q_6 = T$

$$f(u,\ v) = \begin{cases} (x+1, y, l), & \text{if } ((x \ne n) \wedge (l > 0)) \vee ((y = n) \wedge (l \le 0)) \\ (1,\ y+1,\ TIME(y+1) \times v_{y+1}), & \text{if } (x = n) \vee ((l \le 0) \wedge (y \ne n)). \end{cases}$$

The last change in the course of derivation of the value of $\boldsymbol{v_y}$ could occur only in a successful application of a production with the label **5** (if we already generated at least one 1st negation trajectory). Here, after applying the production, $\boldsymbol{v_y}$ was given the value of $\boldsymbol{w_y}$. Consequently, $\boldsymbol{w_y \ne 0}$.

Finally, such a change of the value of $\boldsymbol{w_y}$ for which it would become different from zero, could take place only in a successful application, earlier in the derivation, of one of the productions with the label $\mathbf{4_j}$.

L	Q	Kernel, π_k ($\forall z \in X$)	π_n ($\forall z \in X$)	F_T	F_F
1	Q_1	$S(u, v, w) \to A(u, v, w)$		*two*	$\varnothing$
2_i	Q_2	$A(u, v, w) \to t(h_i^o(u), l_o+1)$ $A((0, 0, 0), g(h_i^o(u),w), zero)$	$TIME(z) = DIST(z,h_i^o(u)$	3	$\varnothing$
3	Q_3	$A(u, v, w) \to A(f(u, v), v, w)$	$NEXTTIME(z) = init(u, NEXTTIME(z))$	*four*	5
4_j	Q_4	$A(u, v, w) \to t(h_j(u), TIME(y)))$ $A(u, v, g(h_j(u), w))$	$NEXTTIME(z) = ALPHA(z, h_j(u), TIME(y) - l+1)$	3	3
5	Q_5	$A(u, v, w) \to A((0, 0, 0), w, zero)$	$TIME(z) = NEXTTIME(z)$	3	6
6	Q_6	$A(u, v, w) \to \varepsilon$		$\varnothing$	$\varnothing$

$$\boldsymbol{g}_{\mathrm{r}}(\mathrm{p_o},\mathrm{t_o},w) = \begin{cases} 1, & \text{if } \boldsymbol{DIST}(\mathrm{r},\mathrm{p_o},\mathrm{t_o}) < 2n, \\ w_{\mathrm{r}}, & \text{if } \boldsymbol{DIST}(\mathrm{r},\mathrm{p_o},\mathrm{t_o}) = 2\mathrm{n}. \end{cases}$$

Let $t_o \in L_t^{l_o}(S)$, $t_o = a(z_o)a(z_1)...a(z_m)$, $t_o \in t_{p_o}(z_o, z_m, m)$;

If $((z_m = y_o) \wedge (p = p_o) \wedge (\exists\, k\ (1 \le k \le m) \wedge (x = z_k))) \vee$
$(((z_m \ne y_o) \vee (p \ne p_o)) \wedge (\exists\, k\ (1 \le k \le m - 1) \wedge (x = z_k)))$
then $DIST(x, p_o, t_o) = k+1$
else $DIST(x, p_o, t_o) = 2n$

$D(\boldsymbol{h}_i) = X \times X \times \mathbf{Z}_+$; Denote $\text{TRACKS} = \bigcup_{ON(p)=x} \text{TRACKS}_p$, where TRACKS_p is the same as for $\boldsymbol{h}_i^o$

If TRACKS = e
then $\boldsymbol{h}_i(u) = e$

else $\text{TRACKS} = \{(p_1,t_1),(p_1,t_2),\ldots,(p_m,t_m)\}$, $(m \le M)$ **and** $\boldsymbol{h}_i(u) = \begin{cases} (p_i,t_i), & \text{if } i \le m, \\ (p_m,t_m), & \text{if } i > m. \end{cases}$

This means that at some stage of derivation symbol $\boldsymbol{t}(\mathbf{p_j}, \mathbf{t_j}, \boldsymbol{\tau}_\mathbf{j})$ was included in the string Z. At the same time, the parameter $\boldsymbol{w}^\mathbf{o} = (\boldsymbol{w}_\mathbf{1}{}^\mathbf{o}, \ldots, \boldsymbol{w}_\mathbf{n}{}^\mathbf{o})$ was changed under the action of the function $\mathbf{g}(\boldsymbol{h}_\mathbf{j}(\boldsymbol{u}), \boldsymbol{w}^\mathbf{o})$ in such a way that $\boldsymbol{w}_\mathbf{y} = \mathbf{g}_\mathbf{y}(\boldsymbol{h}_\mathbf{j}(\boldsymbol{u}), \boldsymbol{w}^\mathbf{o})$.

But $w_y \neq 0$; consequently, $w_y = 1$, i.e., $DIST(y, p_j, t_j) < 2n$, and hence, **y** is included among the parameter values of the t_j trajectory. In addition, obviously, this trajectory is included among the trajectories $t_o, t_1, ..., t_m$, since symbol $t(p_j, t_j, \tau_j)$ was included in **Z** earlier in the course of derivation.

In accord with **Definition of Trajectory Connection**, $\exists\, t_i$ from the set $t_o, t_1, ..., t_m$ such that trajectory t_{m+1} is connected with trajectory t_i, i.e.,

$$C(t_{m+1}, t_i) = T \text{ holds, with } i \leq m.$$

By the assumption of induction

$$C^+{}_{TA(Z)}(t_i, t_o) = T$$

and we conclude that $C^+{}_{TA(Z)}(t_{m+1}, t_o) = T$ (because of the transitivity of C^+).

Thus all the trajectories $t_o, t_1, ..., t_{m+1}$ form a trajectory network.

The theorem is proved.