

Final Exam: April 30 at 8:00 am, Room NC 1313

Final Review: Sample Problems

1. Construct an indexed grammar generating the following language: $\{0^n \mid n \text{ is a power of } 2\}$.
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2. Construct an attribute grammar $G = (V_T, V_N, P, S)$ with the following terminals

$$V_T = \{ce, *, +, (,), \text{ANSWER}_d\},$$

nonterminals

$$V_N = \{E_a, T_b, P_c, S\}$$

and generated attributes **a, b, c, d, e**,
which can compile the following expressions:

$$(ce_1 + ce_2) * (ce_3 + ce_4)$$

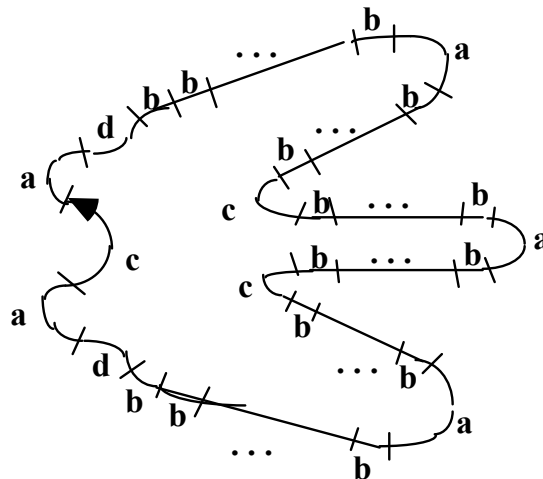
Compile the expression

$$(c_2 + c_5) * (c_{11} + c_3) \text{ANSWER}_d,$$

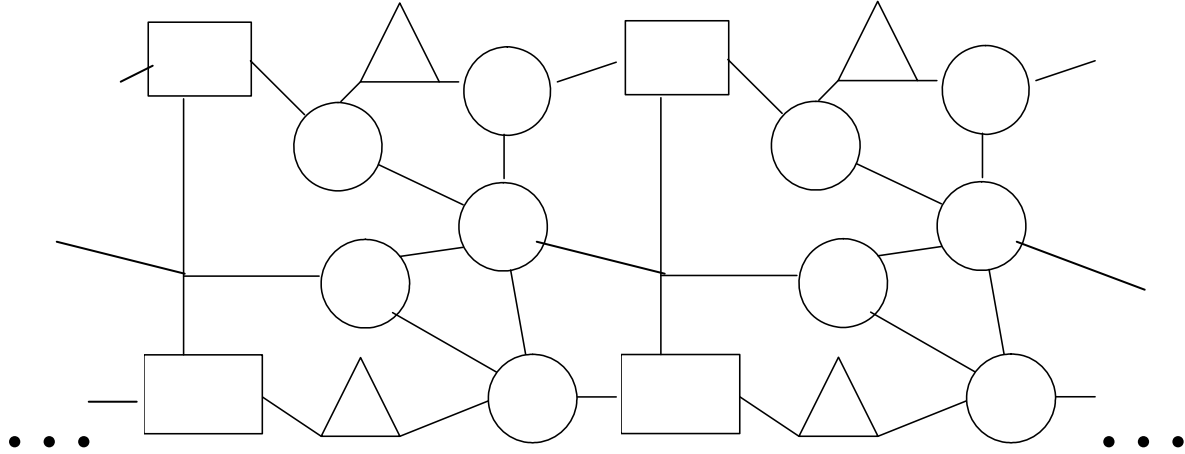
show derivation tree, and find the value of attribute d.

3. Construct a grammar and show a derivation for the mutant chromosome:

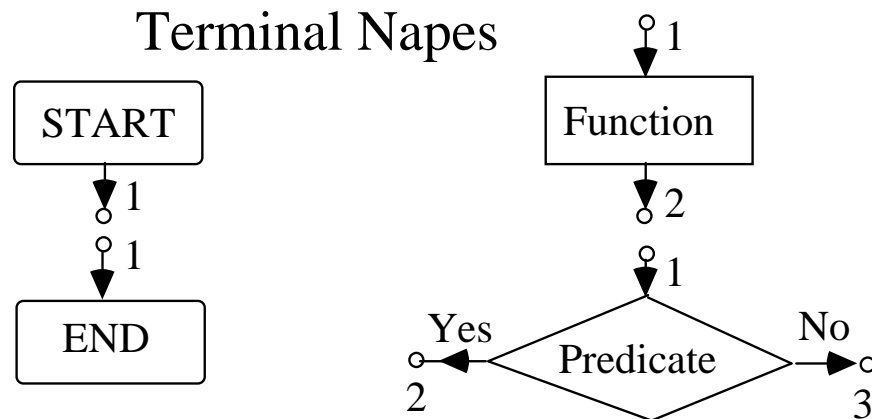
$$V_T = \left\{ \overset{\curvearrowright}{a}, \overset{|}{b}, \overset{\curvearrowleft}{c}, \overset{\curvearrowright}{d} \right\}$$



4. Construct terminal NAPes for generating this kind of networks. They are called Chains of Data Flow Diagrams of arbitrary length. Construct a Plex Grammar generating the following Chain of Data Flow Diagrams (see below). A number of terminal NAPes should be *minimal*.



5. Show all the NAPes which can be generated by the following plex grammar.

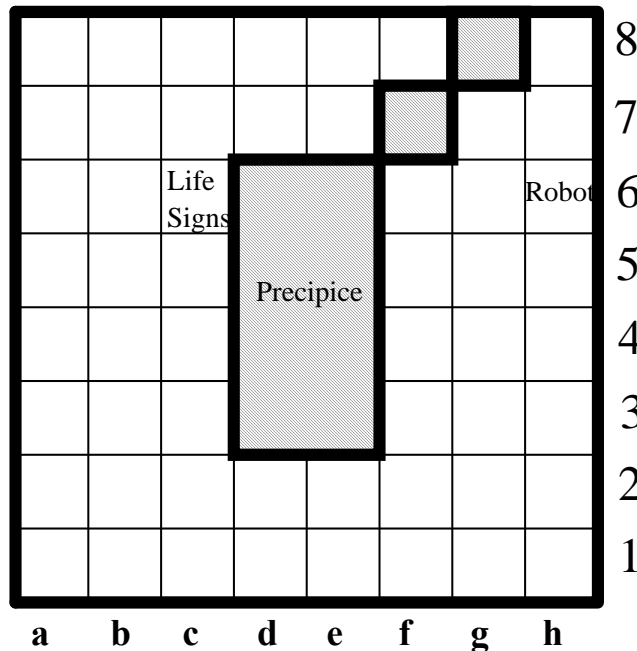


$\langle P \rangle(1,2) \longrightarrow \langle \text{FUNCT} \rangle \langle \text{FUNCT} \rangle(21)(10, 02)$
 $\langle P \rangle(1,2) \longrightarrow \langle \text{PRED} \rangle \langle P \rangle(21, 12)(21, 30)$
 $\langle P \rangle(1,2) \longrightarrow \langle \text{PRED} \rangle \langle P \rangle \langle P \rangle(210, 301, 022)(100, 022)$
 $\langle \text{PROGRAM} \rangle \longrightarrow \langle \text{START} \rangle \langle P \rangle \langle \text{END} \rangle(110, 021)$

6. Generate all the shortest trajectories for the robot called **Bishop** from **a1** to **b6** employing the Grammar of Shortest Trajectories. Show values of all the important functions and sets. (Table TAB15 for Bishop is shown below.)

1		2		2		2		2		2		2		1
	1		2		2		2		2		2		1	
2		1		2		2		2		2		1		2
	2		1		2		2		2		1		2	
2		2		1		2		2		1		2		2
	2		2		1		2		1		2		2	
2		2		2		1		1		2		2		2
	2		2		2		0		2		2		2	
2		2		2		1		1		2		2		2
	2		2		1		2		1		2		2	
2		2		1		2		2		1		2		2
	2		1		2		2		2		1		2	
2		1		2		2		2		2		1		2
	1		2		2		2		2		2		1	
1		2		2		2		2		2		2		1

7. Assume that robot **Explorer** landed on Mars at the square **h6**. It has a map of Mars (shown below) taken from the orbit. It can reach any next square in one step (including diagonal moves). **Explorer** cannot move through the squares, which are in the precipice (shaded), but it can cross the edges of the precipice. The robot received a radio-message from the Earth to investigate all the shortest paths to the location of possible life signs (point **c6**). Its battery has limited capacity and it cannot recharge. Find all these paths employing Grammar of Shortest Trajectories. Show values of all the important functions and sets.



Controlled grammar of shortest trajectories $G_t^{(1)}$

L	Q	Kernel	F_T	F_F
1	Q_1	$S(x,y,l) \rightarrow A(x,y,l)$	<i>two</i>	\emptyset
2_i	Q_2	$A(x,y,l) \rightarrow a(x)A(\mathit{next}_i(x,l), y, f(l))$	<i>two</i>	3
3	Q_3	$A(x,y,l) \rightarrow a(y)$	$\emptyset\emptyset$	

$$V_T = \{a\}$$

$$V_N = \{S, A\}$$

$$V_{PR}$$

$$Pred = \{Q_1, Q_2, Q_3\},$$

$$Q_1(x, y, l) = (\text{MAP}_{x,p}(y) = l) \quad (0 < l < n)$$

$$Q_2(l) = (l \geq 1)$$

$$Q_3 = T$$

$$Var = \{x, y, l\}$$

$$F = \{f, \mathit{next}_1, \dots, \mathit{next}_n\} \quad (n = |X|),$$

$$f(l) = l - 1, \quad D(f) = \mathbf{Z}_+ - \{0\}, \quad \mathit{next}_i \text{ is defined below;}$$

$$E = \mathbf{Z}_+ \cup X \cup P$$

$$\text{Parm: } S \rightarrow Var, \quad A \rightarrow Var, \quad a \rightarrow \{x\}$$

$$L = \{1, 3\} \cup \mathit{two}, \quad \mathit{two} = \{2_1, 2_2, \dots, 2_n\}$$

At the beginning of derivation:

$$x = x_0, \quad y = y_0, \quad l = l_0, \quad x_0 \in X, \quad y_0 \in X, \quad l_0 \in \mathbf{Z}_+, \quad p \in P$$

Function next_i is defined as follows: $D(\mathit{next}_i) = X \times \mathbf{Z}_+ \times X^2 \times \mathbf{Z}_+ \times P$

$\text{MOVE}_l(x)$ is the intersection of the following sets:

$$ST_1(x), \quad ST_{l_0-l+1}(x_0) \quad \text{and} \quad \text{SUM, where}$$

$$\text{SUM} = \{v \mid v \text{ from } X, \text{MAP}_{x_0,p}(v) + \text{MAP}_{y_0,p}(v) = l_0\},$$

$$ST_k(x) = \{v \mid v \text{ from } X, \text{MAP}_{x,p}(v) = k\},$$

If

$$\text{MOVE}_l(x) = \{m_1, m_2, \dots, m_r\} \neq \emptyset$$

then

$$\mathit{next}_i(x, l) = m_i \quad \text{for } i \leq r;$$

$$\mathit{next}_i(x, l) = m_r \quad \text{for } r < i \leq n,$$

otherwise

$$\mathit{next}_i(x, l) = x.$$

Sample Solutions

1. $G = \{V_T, V_N, F, P, S\}$

$$V_T = \{\mathbf{0}\}, V_N = \{S, A, B\}$$

$$F = \{\mathbf{a}, \mathbf{b}\}$$

P: 1. $S \longrightarrow Ab$

2. $A \longrightarrow Aa$

$$\mathbf{a} = \{A \longrightarrow BB, B \longrightarrow BB\}$$

$$\mathbf{b} = \{A \longrightarrow \mathbf{0}, B \longrightarrow \mathbf{0}\}$$

Derivation

$$n = 0 \quad S \stackrel{1}{\Rightarrow} Ab \stackrel{b_1}{\Rightarrow} \mathbf{0}$$

$$n = 1 \quad S \stackrel{1}{\Rightarrow} Ab \stackrel{2}{\Rightarrow} Aab \stackrel{a_1}{\Rightarrow} BbBb \stackrel{b_2 \times 2}{\Rightarrow} \mathbf{0}^2$$

$$n > 1 \quad S \stackrel{1}{\Rightarrow} Ab$$

$$\left. \begin{array}{l} \stackrel{2}{\Rightarrow} Aab \\ \stackrel{2}{\Rightarrow} Aa^2b \\ \dots\dots \end{array} \right\} n \text{ times}$$

$$\left. \begin{array}{l} \stackrel{2}{\Rightarrow} Aa^n b \\ \stackrel{a_1}{\Rightarrow} Ba^{n-1} b Ba^{n-1} b \\ \stackrel{a_2 \times 2}{\Rightarrow} \underline{Ba^{n-2} b B a^{n-2} b} \quad \underline{Ba^{n-2} b B a^{n-2} b} \\ \stackrel{a_2 \times 2^2}{\Rightarrow} \dots \\ \dots\dots \\ \stackrel{a_2 \times 2^{n-1}}{\Rightarrow} (Bb)^{2^n} \end{array} \right\} n \text{ times}$$

$$\left. \begin{array}{l} \stackrel{b_2}{\Rightarrow} \mathbf{0}(Bb)^{2^n-1} \\ \dots\dots \\ \stackrel{b_2}{\Rightarrow} \mathbf{0}^{2^n} \end{array} \right\} 2^n \text{ times}$$

2. 1) $S \rightarrow E_a \text{ ANSWER } b$ $b := a$
 2) $E_d \rightarrow E_a + T_f$ $d := e + f$
 3) $E_g \rightarrow T_h$ $g := h$
 4) $T_i \rightarrow T_j * P_k$ $i := j * k$
 5) $T_m \rightarrow P_n$ $m := n$
 6) $P_p \rightarrow (E_q)$ $p := q$
 7) $P_r \rightarrow c_s$ $r := s$

