Assignment 9. Due **04/11/11**

1. Consider the language $\{0^n1^n0^n, n = 1, 2, 3, \ldots\}$, and **programmed grammar**:

label	kernel	F_S	F_F
1	S —> ABC	2,5	∅
2	A —> 0A	3	∅
3	B —> 1B	4	∅
4	C —> 0C	2,5	∅
5	A —> 0	6	∅
6	B —> 1	7	∅
7	C —> 0	∅	∅

Show derivation of the string $0^n1^n0^n$.

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2. Consider the language from the problem #1. Construct an **indexed** grammar, generating this language, and show the derivation. (Indexed grammars will be considered in the next handout.)

3. Consider language $L = \{a^nb^nc^nd^n, n \geq 1\}$. Construct a context-free **programmed** grammar, generating this language, and show the derivation.

4-5. **Extra credits.**

Complete derivations for the general case for
Example 5 (this handout) and
Example 2 (next handout)
considered in class.

Programmed Grammars (Rosenkrantz)

$$G = (V_T, V_N, P, L, S)$$

V_T - terminal symbols
V_N - nonterminal symbols
S - start symbol
L - finite set of "labels"
P is a finite set of productions:

$$(l) \quad \alpha \longrightarrow \beta \ (F_S)\ (F_F),$$

where

l is a label of the production, $l \in L$

$\alpha \longrightarrow \beta$ kernel of the production

$\alpha \in (V_T \cup V_N)^* V_N (V_T \cup V_N)^*$,
$\beta \in (V_T \cup V_N)^*$;
(this is an ordinary production)

F_S is a finite set of transitions in case of <u>**s**uccess</u> $F_S \subseteq L$

F_F is a finite set of transitions in case of <u>failure</u> $F_F \subseteq L$

<u>Types of Programmed Grammars:</u>

Type of a kernel defines the type of the grammar.
Types: 1, 2, 3.

For example: $A \longrightarrow \beta$, $A \in V_N$, $\beta \in (V_T \cup V_N)^+$
<u>e-free, context-free.</u>

Derivation in Programmed Grammars

Let $\beta \in V_T^*$ and denote

$$S \underset{(l_1, \ldots, l_n)}{\Rightarrow^*} \beta \text{ , if}$$

$$S \underset{l_1}{\Rightarrow} \gamma_1 \underset{l_2}{\Rightarrow} \gamma_2 \underset{l_3}{\Rightarrow} \ldots \underset{l_n}{\Rightarrow} \gamma_n = \beta$$

<u>1st step:</u>

(l_1) $S \longrightarrow \gamma_1$ is applied to S

. . . .

<u>i-th step:</u>

(l_i) $\alpha_i \longrightarrow \beta_i$ (F_{S_i}) (F_{F_i})

If production with label l_i can be applied then (l_i) $\alpha_i \longrightarrow \beta_i$ is applied to the string $\gamma_{i-1} \in (V_T \cup V_N)^* \alpha_i (V_T \cup V_N)^*$; the leftmost instance of symbol α_i is replaced with β_i, then <u>go to</u> the production with label $l_{i+1} \in F_{S_i}$. If $F_{S_i} = \emptyset$ the derivation stops.

If production with label l_i cannot be applied, <u>go to</u> the production with label $l_{i+1} \in F_{F_i}$. (It means there is no instance of α_i in the string γ_{i-1}.) If $F_{F_i} = \emptyset$ the derivation stops.

Examples

1. Programmed Grammar

$G = (V_T, V_N, P, L, S)$

$V_N = \{S, B, C\}$

$V_T = \{a, b, c\}$

$L = \{1, 2, 3, 4, 5\}$

P:

l	Kernel	F_S	F_F
1	S —> aB	(2,3,6)	∅
2	B —> aBB	3	∅
3	B —> C	4	5
4	C —> bC	3	∅
5	C —> c	5	∅
6	B —> aaBBB	3	∅

Derivation for $a^3b^3c^3$:

$$S \overset{1}{\Rightarrow} aB \overset{6}{\Rightarrow} aaaBBB$$

$$\overset{3}{\Rightarrow} a^3CB^2 \overset{4}{\Rightarrow} a^3bCB^2 \overset{3}{\Rightarrow} a^3bC^2B \overset{4}{\Rightarrow} a^3b^2C^2B$$

$$\overset{3}{\Rightarrow} a^3b^2C^3 \overset{4}{\Rightarrow} a^3b^3C^3 \overset{5}{\Rightarrow} a^3b^3\mathbf{c}C^2 \overset{5}{\Rightarrow} a^3b^3\mathbf{c}^2C \overset{5}{\Rightarrow} a^3b^3\mathbf{c}^3$$

$L = \{a^nb^nc^n, 1 \le n \le 3\}$

2. **Context-free Chomsky grammar**

$G_2 = (V_T, V_N, P, S)$

$V_T = \{a, b, c\}$

$V_N = \{S, A_1, A_2, B_1, B_2, B_3, C\}$

1) $S \longrightarrow aA_1C$	4) $A_1 \longrightarrow aA_2C$	7) $B_2 \longrightarrow bB_1$
2) $A_1 \longrightarrow b$	5) $A_2 \longrightarrow aB_3C$	8) $B_1 \longrightarrow b$
3) $A_1 \longrightarrow aB_2C$	6) $B_3 \longrightarrow bB_2$	9) $C \longrightarrow c$

Show the derivation for $a^3b^3c^3$:

$$S \xrightarrow{1} aA_1C \xrightarrow{4} aaA_2C^2 \xrightarrow{5} a^3B_3C^3$$

$$\xrightarrow{6} a^3bB_2C^3 \xrightarrow{7} a^3bbB_1C^3 \xrightarrow{8} a^3b^3C^3$$

$$\xrightarrow{9} a^3b^3C^2c \xrightarrow{9} a^3b^3Cc^2 \xrightarrow{9} a^3b^3c^3$$

$L = \{a^nb^nc^n, 1 \le n \le 3\}$ the same language as in Example 1.

Regular grammar for the same language:

1) $S \longrightarrow aA_1$	6) $B_{10} \longrightarrow bC_1$	11) $B_{32} \longrightarrow bC_3$
2) $S \longrightarrow aB_{10}$	7) $B_{20} \longrightarrow bB_{21}$	12) $C_1 \longrightarrow c$
3) $A_1 \longrightarrow aA_2$	8) $B_{21} \longrightarrow bC_2$	13) $C_2 \longrightarrow cC_1$
4) $A_1 \longrightarrow aB_{20}$	9) $B_{30} \longrightarrow bB_{31}$	**14)** $C_3 \longrightarrow cC_2$
5) $A_2 \longrightarrow aB_{30}$	10) $B_{31} \longrightarrow bB_{32}$	

Show the derivation for $a^3b^3c^3$:

$$S \xrightarrow{1} aA_1 \xrightarrow{3} aaA_2 \xrightarrow{5} a^3B_{30} \xrightarrow{9} a^3bB_{31} \xrightarrow{10} a^3b^2B_{32}$$

$$\xrightarrow{11} a^3b^3C_3 \xrightarrow{14} a^3b^3cC_2 \xrightarrow{13} a^3b^3c^2C_1 \xrightarrow{12} a^3b^3c^3$$

3. Consider another **programmed grammar**:

$G = (V_T, V_N, P, L, S)$

$V_T = \{a, b, c\}$

$V_N = \{S, R, T\}$

$L = \{1, 2, 3, 4, 5\}$

P is as follows:

ℓ	Kernel (context-free)	F_S	F_F
1	S —> RT	(2, 3)	∅
2	R —> aR	4	∅
3	R —> a	5	∅
4	T —> bTc	(2, 3)	∅
5	T —> bc	∅	∅

Show derivation of the string $a^n b^n c^n$:

Derivation for aabbcc:

S => RT =>(aR)T=> aR(bTc) => a(a)bTc => aab(bc)c = $a^2b^2c^2$

For $a^n b^n c^n$:

$$S \overset{1}{\Rightarrow} RT$$

$$n-1 \left\{ \begin{array}{l} \overset{2}{\Rightarrow} aRT \overset{4}{\Rightarrow} aRbTc \\ \overset{2}{\Rightarrow} aaRbTc \overset{4}{\Rightarrow} aaRbbTcc \\ \quad \cdots\cdots\cdots \\ \overset{2}{\Rightarrow} a^{n-1}Rb^{n-2}Tc^{n-2} \overset{4}{\Rightarrow} a^{n-1}R\,b^{n-1}Tc^{n-1} \end{array} \right.$$

$$\overset{3}{\Rightarrow} a^{n-1}ab^{n-1}Tc^{n-1} \overset{5}{\Rightarrow} a^n b^n c^n \quad (n \geq 1)$$

Run time: 2n + 1 steps

We know that this is a context-sensitive language.

A Chomsky grammar for this language (see previous class):

$G = (V_T, V_N, P, S)$

$V_T = \{a, b, c\}$

$V_N = \{S, R, T\}$

P:
1) S—> aSRT
2) S—> aRT
3) TR—> RT
4) aR—> ab
5) bR—> bb
6) bT—> bc
7) cT—> cc

It is a context-sensitive grammar.

Run time: n(n+5)/2.

4. Consider different **programmed grammar**:

$G = (V_T, V_N, P, L, S)$

$V_T = \{a, b, c\}$ $V_N = \{A, B, C\}$ $L = \{1, 2, 3, 4, 5\}$

ℓ	Kernel	F_S	F_F
1	$S \longrightarrow aBC$	2,4	$\varnothing$
2	$B \longrightarrow aBB$	3	$\varnothing$
3	$C \longrightarrow CC$	2,4	$\varnothing$
4	$B \longrightarrow b$	4	5
5	$C \longrightarrow c$	5	$\varnothing$

What language does it generate?

Derivation for $a^2b^2c^2$:

$$S \xrightarrow{1} aBC \xrightarrow{2} aaBBC \xrightarrow{3} aaBBCC \xrightarrow{4} aabBCC \xrightarrow{4} a^2b^2C^2$$

$$\xrightarrow{5} a^2b^2cC \xrightarrow{5} a^2b^2c^2.$$

Consider derivation for $a^nb^nc^n$:

$$S \xrightarrow{1} aBC$$

$$n-1 \left\{ \begin{array}{l} \xrightarrow{2} a^2B^2C \xrightarrow{3} a^2B^2C^2 \\ \xrightarrow{2} a^3B^3C^2 \xrightarrow{3} a^3B^3C^3 \\ \ldots\ldots\ldots\ldots\ldots\ldots \end{array} \right.$$

$$\xrightarrow{2} a^nB^nC^{n-1} \xrightarrow{3} a^nB^nC^n$$

$$n \left\{ \begin{array}{l} \xrightarrow{4} a^nbB^{n-1}C^n \\ \ldots\ldots \\ \xrightarrow{4} a^nb^nC^n \end{array} \right.$$

$$\xrightarrow{5} a^nb^ncC^{n-1} \xrightarrow{5} \ldots \xrightarrow{5} a^nb^nc^n$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{n}$$

Total = 4n-1 steps

5. Consider a Chomsky grammar

$G = (V_T, V_N, P, S)$

$V_T = \{a, b, c, d\}$

$V_N = \{S, A, B, C, D, E, F, G\}$

P:

1) S —> aAB
2) A —> aAC
3) A —> D
4) Dc —> cD
5) Dd —> dD
6) DC —> EC
7) EC —> Ed
8) DB —> FB
9) Ed —> Gd
10) cG —> Gc
11) dG —> Gd
12) aG —> abcD
13) bG —> bbcD
14) dFB —> dFd
15) dFd —> Fdd
16) cF —> Fc
17) bF —> bbc
18) aF —> ab
19) bB —> bcd

It generates $L = \{ a^n b^n c^n d^n \mid n \geq 1\}$

(See assignments)