Assignment 8. Due 04/04/11.

1. Complete Backus-Naur definition of Pascal expressions (pp. 7-9 of this handout).

2. Consider a grammar $G = (V_T, V_N, P, S)$

   $V_T = \{a, b\}$

   $V_N = S$

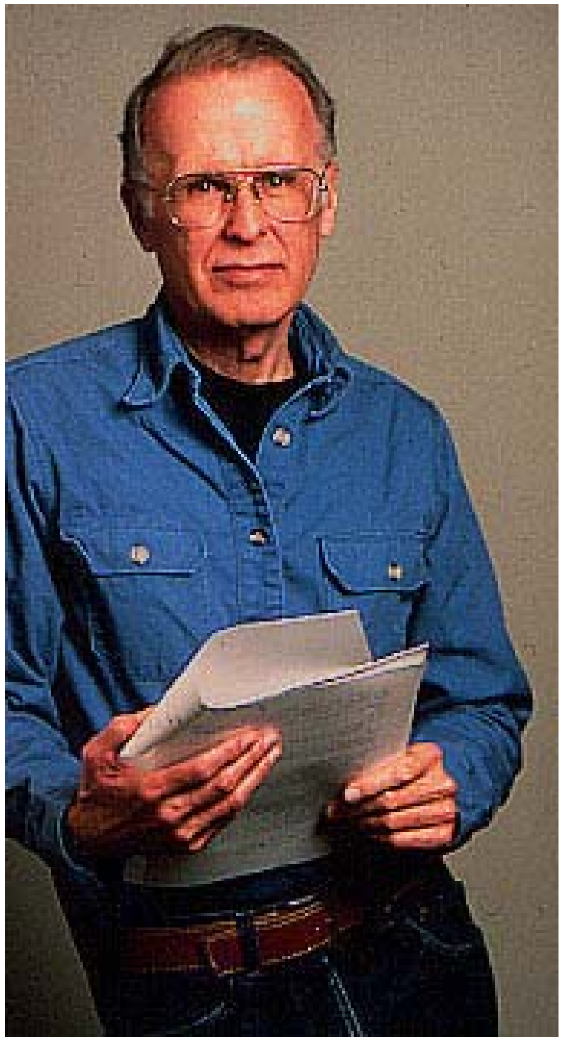
   P:

   S —> aSb,

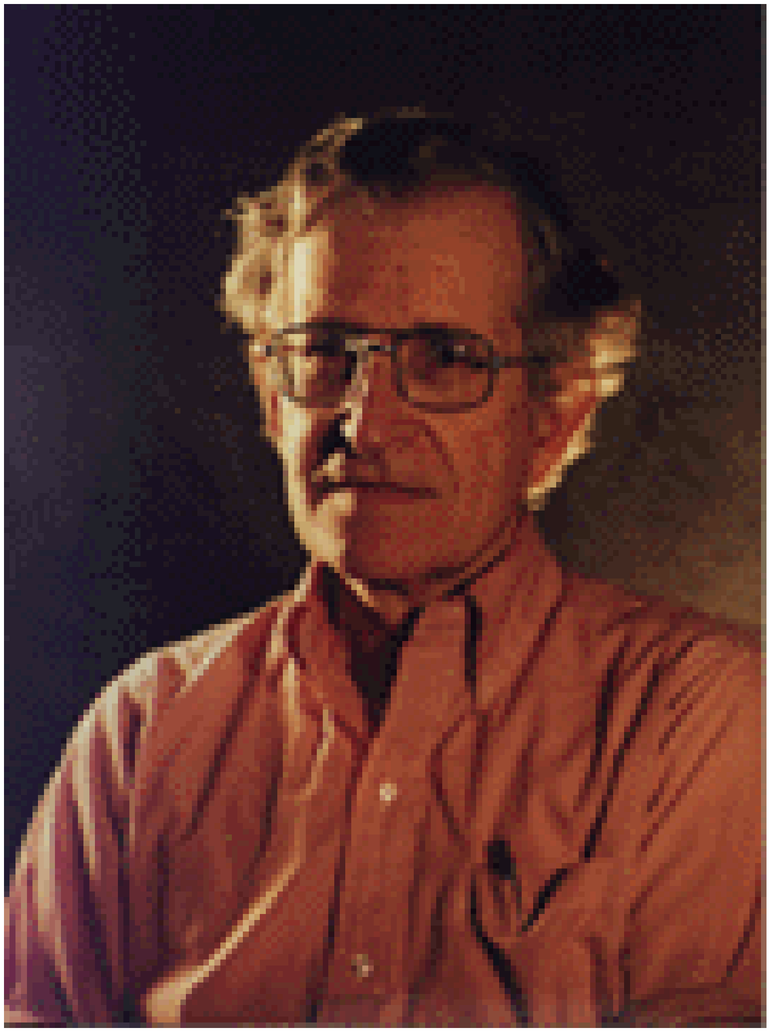
   S —> ab

Find the language generated by this grammar.

Prove that this is the only language, which can be generated by this grammar.



John Backus



NOAM CHOMSKY

Formal Grammars

Let us parse the sentence:

The girl walks gracefully.
======= =============
noun phrase *verb phrase*

1. <sentence>
2. <noun phrase> <verb phrase>
3. <adjective> <noun> <verb phrase>
4. The <noun> <verb phrase>
5. The girl <verb phrase>
6. The girl <verb> <adverb>
7. The girl walks gracefully.

We can use rules:

<sentence> —> <noun phrase> <verb phrase>
<noun phrase> —> <adjective> <noun>
<verb phrase> —> <verb> <adverb>
<adjective> —> the
<noun> —> girl
<verb> —> walks
<adverb> —> gracefully

Symbol —> means "may be replaced"

Formal Grammars

Definition:

A generating grammar is the following four-tuple:

$$G = (V_{\mathbf{T}}, V_{\mathbf{N}}, P, S),$$

where

$V_{\mathbf{T}}$ is a finite set of terminal symbols

$V_{\mathbf{N}}$ is a finite set of nonterminal symbols (or variables)

P is a finite set of rules (productions) of the form:

$$\alpha \longrightarrow \beta,$$

where α and β are strings of symbols from V:

$$V = V_{\mathbf{T}} \cup V_{\mathbf{N}}$$

$$V_{\mathbf{T}} \cap V_{\mathbf{N}} = \emptyset,$$

α consists of at least one symbol of $V_{\mathbf{N}}$

Usually they write:

$$\alpha, \beta \in V^*$$

S is a special nonterminal symbol called start symbol, $S \in V_{\mathbf{N}}$.

In our example

$V_{\mathbf{T}}$ = {The, girl, walks, gracefully}

$V_{\mathbf{N}}$ = {<sentence>, <noun phrase>, <verb phrase>,
<adjective>, <noun>, <verb>, <adverb>}

S= <sentence>

What about the sentence:

They are rocking chairs.

Formal Grammars

Definition of the language generated by the grammar

$$G = (V_T, V_N, P, S)$$

Let us define two relations $\underset{G}{=>}$ and $\underset{G}{=>^*}$ between strings from $(V_T \cup V_N)^*$.

Let $\gamma_1\alpha\gamma_2 \in (V_T \cup V_N)^+ \subset (V_T \cup V_N)^*$, i.e., is a string of terminal and nonterminal symbols of the length ≥ 1.

If $\alpha \longrightarrow \beta$ is a production from P, then substring α of the string $\gamma_1 \alpha\gamma_2$ can be replaced by the string β, and the result will be: $\gamma_1\beta\gamma_2$

They write as follows: $\gamma_1 \alpha\gamma_2 => \gamma_1\beta\gamma_2$

and say, that $\gamma_1 \alpha\gamma_2$ generates the string $\gamma_1\beta\gamma_2$.

If $\alpha_1, \alpha_2, \ldots, \alpha_n \in (V_T \cup V_N)^*$

and $\alpha_1 \underset{G}{=>} \alpha_2$, $\alpha_2 \underset{G}{=>} \alpha_3$, … , $\alpha_{n-1} \underset{G}{=>} \alpha_n$ $(n \geq 1)$

then usually they write:

$\alpha_1 \underset{G}{=>} \alpha_2 \underset{G}{=>} \alpha_3 \underset{G}{=>} \ldots \underset{G}{=>} \alpha_{n-1} \underset{G}{=>} \alpha_n$, or

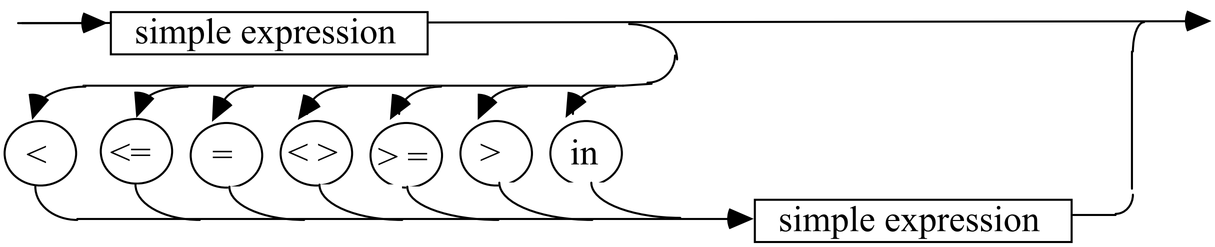
$\alpha_1 \underset{G}{=>^*} \alpha_n$

The language generated by G is the following set:

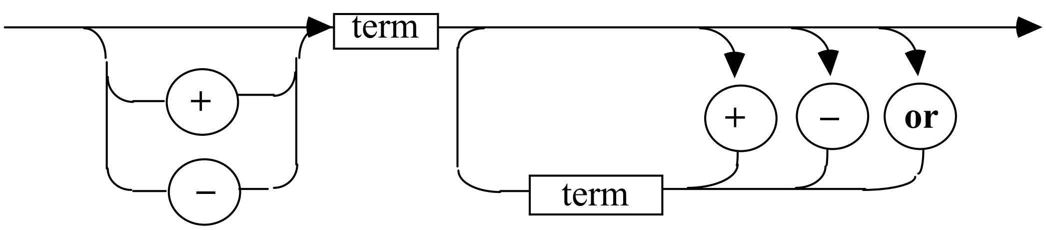
$$L(G) = \{\alpha \mid \alpha \in V_T^* \text{ and } S =>^* \alpha\}$$

Examples of Grammars

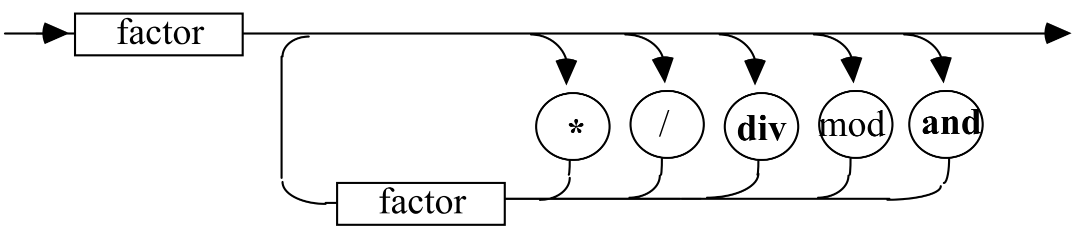
Expression



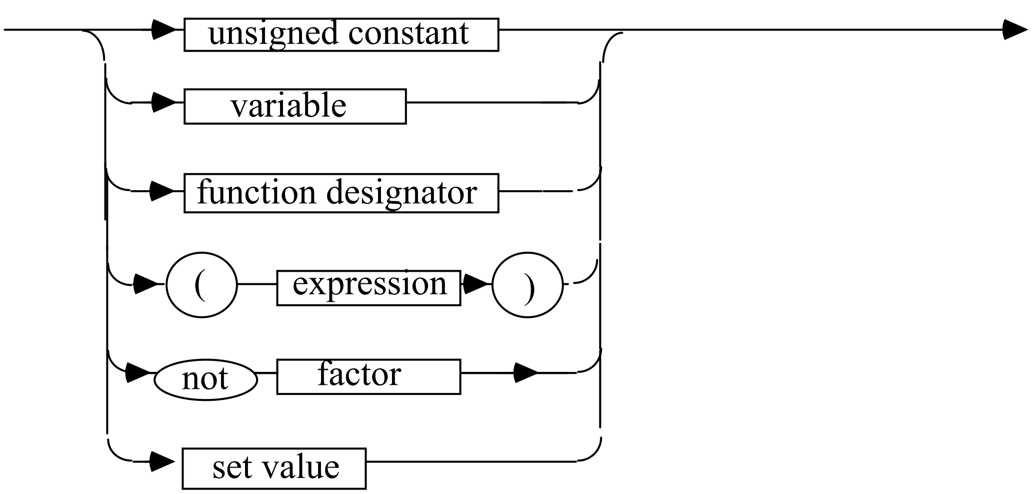
Simple Expression



Term

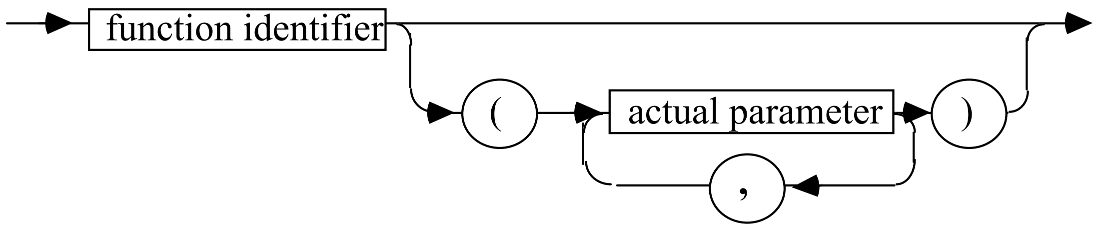


Factor

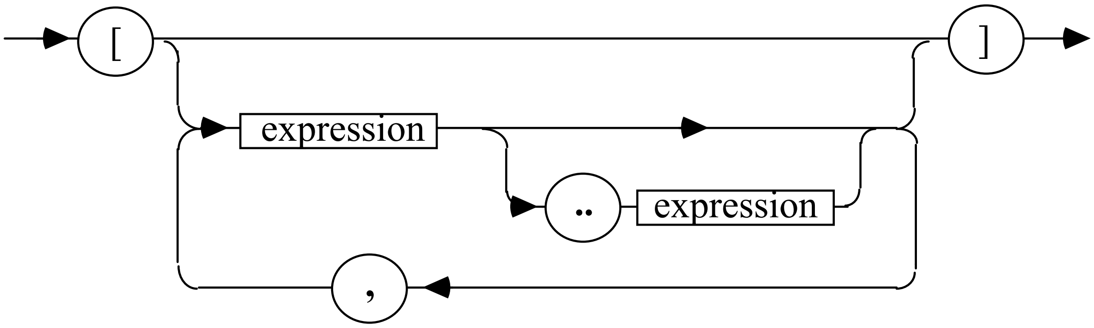


Examples of Grammars

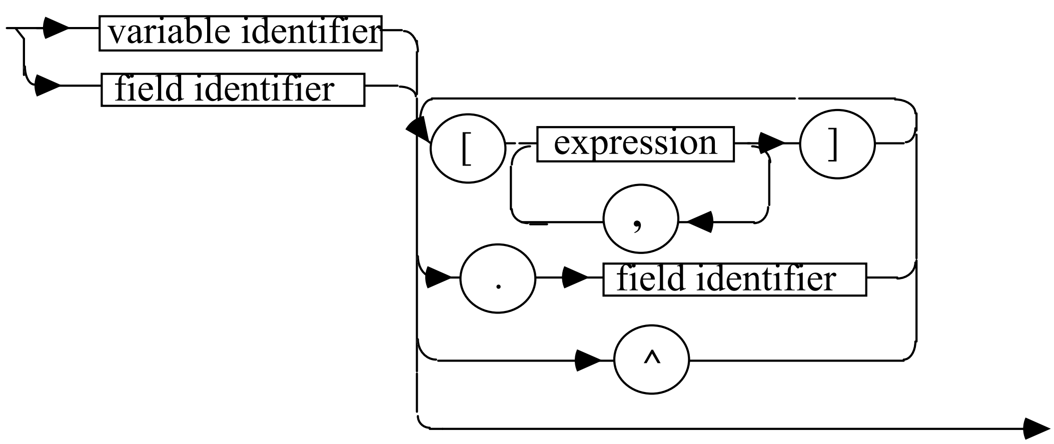
Function Designator



Set Value

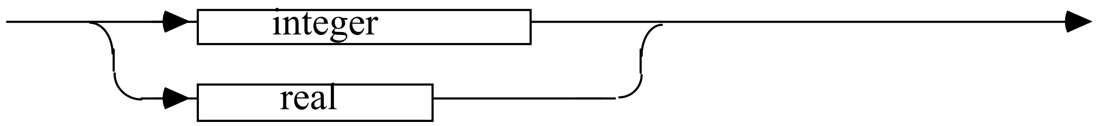


Variable

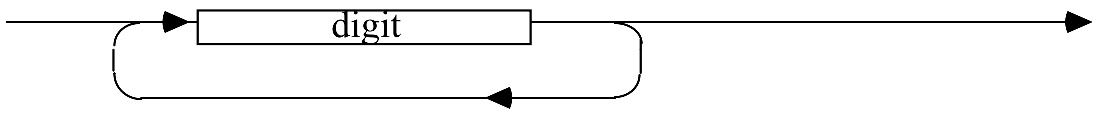


Examples of Grammars

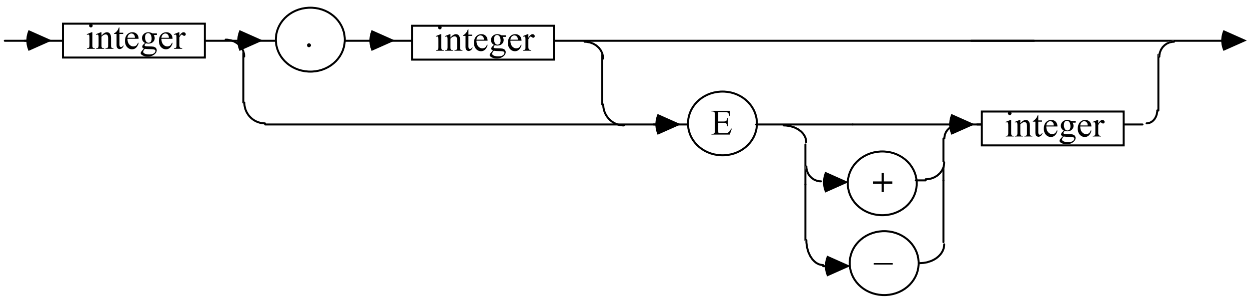
Unsigned Number



Integer



Real



Backus-Naur Form (BNF)

Sample definitions

<expression> —>

<simple expression>

| <simple expression> <relation> <simple expression>

<relation> —> < | <= | = | < > | >= | > | in

<simple expression> —> <sign> <term> <term tail>

<sign> —> λ | + | –

<term tail> —> λ | <operation> <term><term tail> *(recursive production)*

<operation> —> + | – | or

Examples of Grammars

$$G = (V_{\mathbf{T}}, V_{\mathbf{N}}, P, S)$$

$V_{\mathbf{T}} = \{a, b, c\}$

$V_{\mathbf{N}} = \{S, B, C\}$

P :	1. S—>aSBC	4. aB—>ab
	2. S—>aBC	5. bB—>bb
	3. CB—>BC	6. bC—>bc
		7. cC—>cc

Let us derive $a^{2}b^{2}c^{2}$:

$$S \xrightarrow{1} aSBC \xrightarrow{2} aaBCB \xrightarrow{4} aabCBC \xrightarrow{3} aabBCC \xrightarrow{5}$$

$$aabbCC \xrightarrow{6} aabbcC \xrightarrow{7} aabbcc$$

We can prove that we can derive any string of the form:
$a^{n}b^{n}c^{n}$, $n \geq 1$.