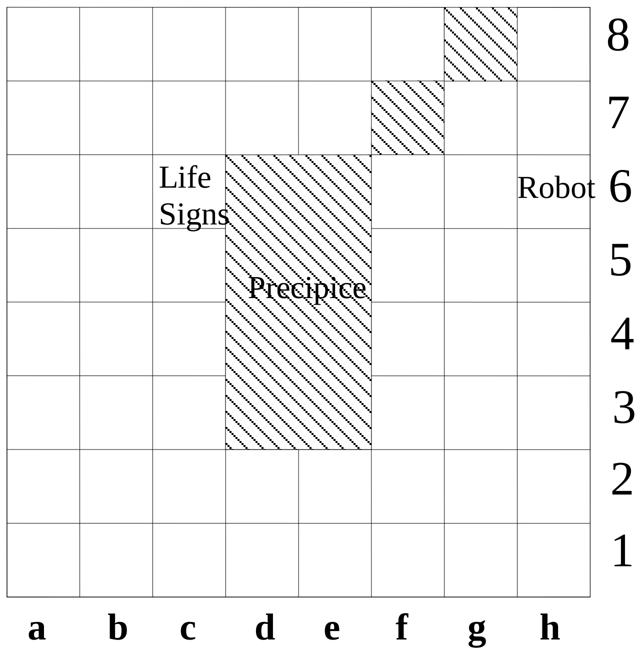
MIDTERM REVIEW: SAMPLE PROBLEMS

1. Assume that robot **Explorer** landed on Mars at the square **h6**. It has a map of Mars (shown below) taken from the orbit. It can reach any next square in one step (including diagonal moves). **Explorer** cannot move through the squares, which are in the precipice (shaded), but it can cross the edges of the precipice. The robot received a radio-message from the Earth to investigate **all** the shortest paths to the location of possible life signs (point **c6**). Find **all** these paths. Show a search tree, explain your search algorithm, heuristics.



2. Two players p_1 and p_2 play a game. In the state s_1 player p_1 is either at the point **a**, or at the point **b**. From the state s_1 p_2 can get into any point. Motion of the player p_2 does not change the location of player p_1 (frame axiom). Does the state s_3 exist (or a set of states) such that p_1 and p_2 are at the same point? Answer this question by resolution (employing variable s).

3. Consider the following story of the lucky student. Anyone passing their history exams and winning the lottery is happy. But anyone who studies or is lucky can pass all their exams. John did not study but he is lucky. Anyone who is lucky wins the lottery. Is John happy?

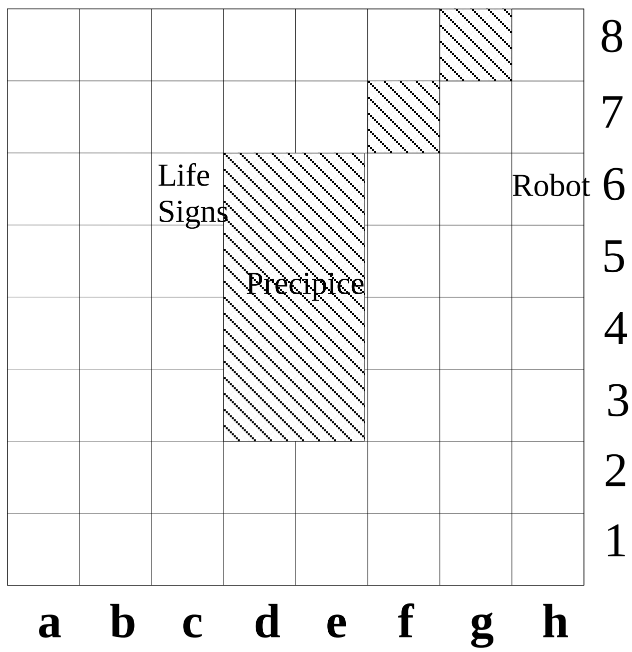
4. All people that are not poor and are smart are happy. Those people that read are not stupid. John can read and is wealthy. Happy people have exciting lives. Can anyone be found with an exciting life?

5. Consider the game of tic-tac-toe. Find the winning strategy employing MINIMAX algorithm with unlimited depth. Show the search tree of unlimited depth, minimax values, cut-offs, and the optimal branch(es). What heuristic function would you use?

Solutions to Several Problems

(1, 2, 3, 4)

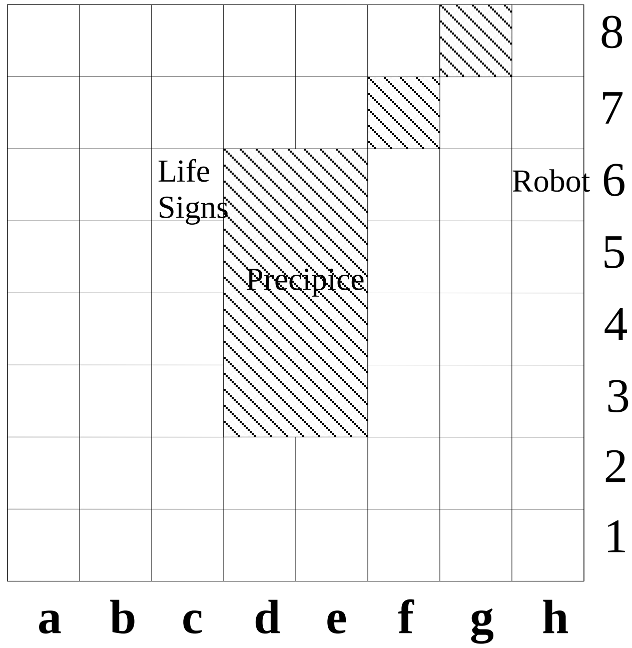
1. Assume that robot **Explorer** landed on Mars at the square h6. It has a map of Mars (shown below) taken from the orbit. It can reach any next square in one step (including diagonal moves). **Explorer** cannot move through the squares, which are in the precipice (shaded), but it can cross the edges of the precipice. The robot received a radio-message from the Earth to investigate **all** the shortest paths to the location of possible life signs (point c6). Its battery has limited capacity and it cannot recharge. Find **all** these paths. Show a search tree, explain your algorithm, heuristics.



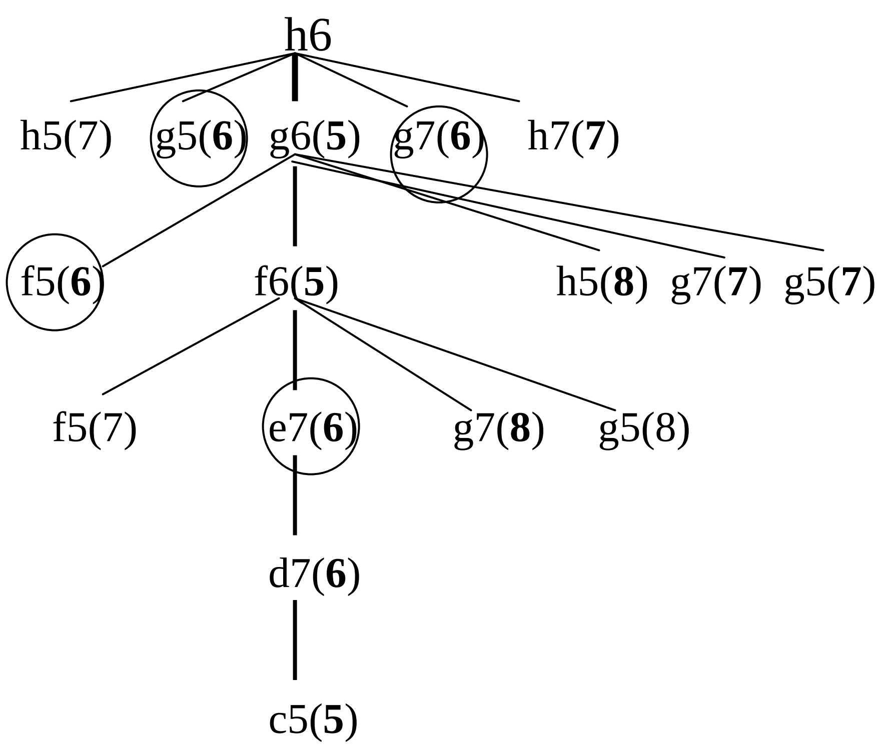
Mars

This is not a complete solution. Let us apply an A* search algorithm. The value of the heuristic function **h** will be the "distance" to the location of "life signs". An example of such function could be a sum of horizontal plus vertical distances, e.g., **h**(f6) = 3 + 0 = 3. The quality of our solution depends on the quality of the function measuring distance. Obviously, our function **h** is not a very good function because **h**(f8) = 3 + 2 = 5 while from f8 robot can get to c6 in the same number of steps as from f6.

As usual, the total value of the evaluation function for A* (**f** = **h** + **g**) is the sum of heuristic function **h**, an estimated distance to destination, plus **g**, the depth of the node in the search tree (a distance from the start state).



Mars



After f6-e7 all the positions in the bubbles are evaluated as 6, which means that they would become candidates for expansion. Eventually, this algorithm will find all the shortest routes but after exploring many non-shortest routes, thus, robot's battery may discharge and **Explorer** may not complete its task.

2. Two players p_1 and p_2 play a game. In the state s_1 player p_1 is either at the point **a**, or at the point **b**. From the state s_1 p_2 can get into any point. Motion of the player p_2 does not change the location of the player p_1. Does the state s_3 exist (or a set of states) such that p_1 and p_2 are at the same point?

Operator: move(p, x, s)

State space $\{AT(p_1, x, s) \wedge AT(p_2, y, s)\}$

Formulas:

1) $AT(p_1, a, s_1) \vee AT(p_1, b, s_1)$
2) $\forall y_2\ AT(p_2, y_2, move(p_2, y_2, s_1))$
3) $\forall x\ \forall y\ \forall s\ AT(p_1, x, s) \Rightarrow AT(p_1, x, move(p_2, y, s))$

Question: $\exists x_3\ \exists s_3\ AT(p_1, x_3, s_3) \wedge AT(p_2, x_3, s_3)$

Clauses:

1) $AT(p_1, a, s_1) \vee AT(p_1, b, s_1)$
2) $AT(p_2, y_2, move(p_2, y_2, s_1))$
3) $\neg AT(p_1, x, s) \vee AT(p_1, x, move(p_2, y, s))$

Negation of the Question $\vee$ **ANSWER**(x_3, s_3)

$\neg AT(p_1, x_3, s_3) \vee \neg AT(p_2, x_3, s_3) \vee$ **ANSWER**(x_3, s_3)

2

$(move(p_2, y_2, s_1)/s_3)\ (y_2/x_3)$

$\neg AT(p_1, y_2, move(p_2, y_2, s_1)) \vee$ **ANSWER**$(y_2, move(p_2, y_2, s_1))$

3

$(y_2/x)\ (y_2/y)\ (s_1/s)$

(*) $\neg AT(p_1, y_2, s_1) \vee$ **ANSWER**$(y_2, move(p_2, y_2, s_1))$

1

(a/y_2)

$AT(p_1, b, s_1) \vee$ **ANSWER**$(a, move(p_2, a, s_1))$

(*)

Reintroduce (*) with y_2'and s_1'; $(b/y_2')\ (s_1/ s_1')$

ANSWER$(a, move(p_2, a, s_1)) \vee$ **ANSWER**$(b, move(p_2, b, s_1))$

3.

Consider the following story of the "lucky student":

Anyone passing their history exams and winning the lottery is happy. But anyone who studies or is lucky can pass all their exams. John did not study but he is lucky. Anyone who is lucky wins the lottery. Is John happy?

First change the sentences to predicate form:

Anyone passing their history exams and winning the lottery is happy.

$\forall X(\text{pass}(X, \text{history}) \wedge \text{win}(X, \text{lottery}) \rightarrow \text{happy}(X))$

Anyone who studies or is lucky can pass all their exams.

$\forall X \forall Y (\text{studies}(X) \vee \text{lucky}(X) \rightarrow \text{pass}(X,Y))$

John did not study but he is lucky.

$\neg \text{study}(\text{john}) \wedge \text{lucky}(\text{john})$

Anyone who is lucky wins the lottery.

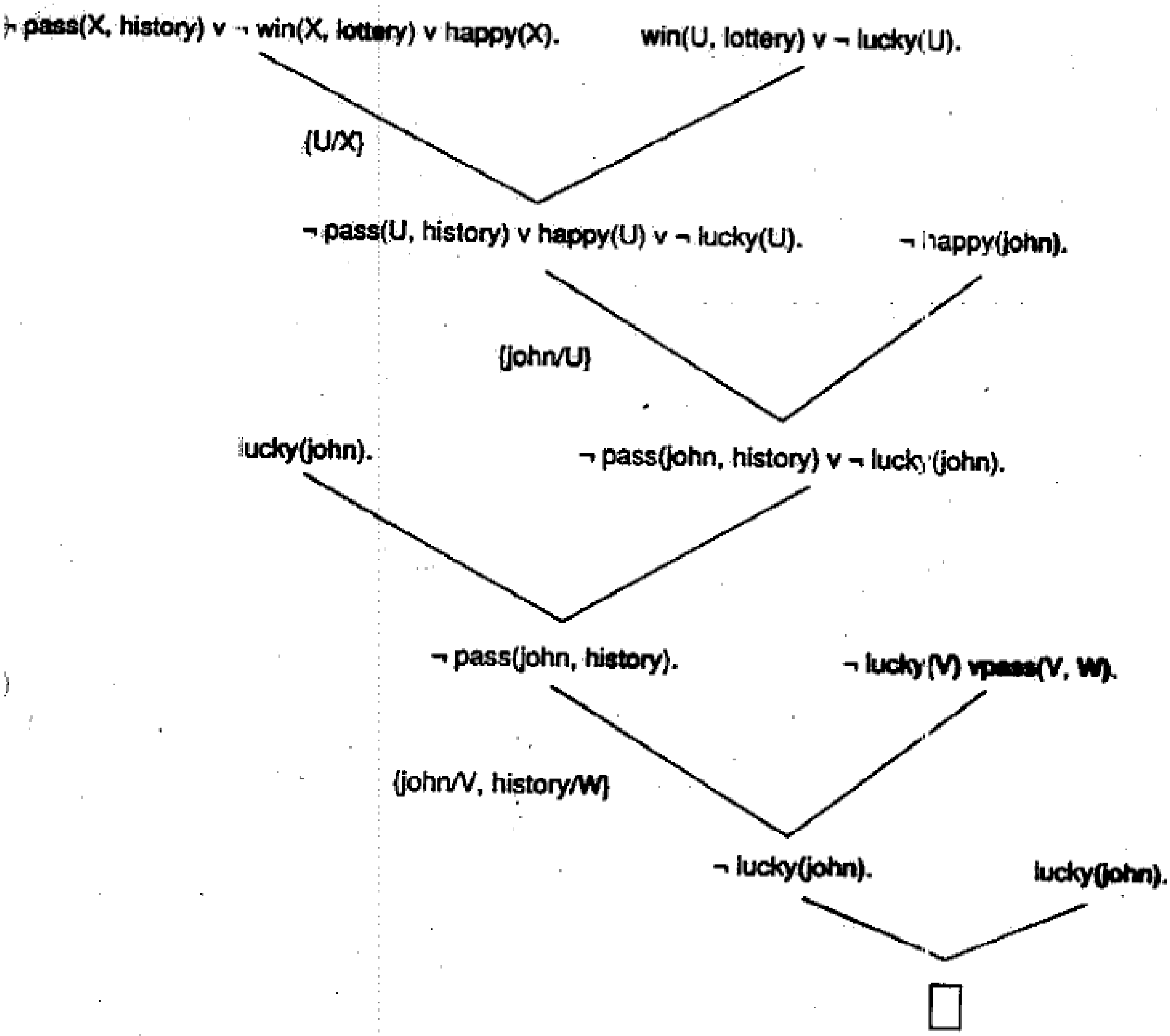
$\forall X (\text{lucky}(X) \rightarrow \text{win}(X, \text{lottery}))$

These four predicate statements are now changed to clause form

1. $\neg \text{pass}(X, \text{history}) \vee \neg \text{win}(X, \text{lottery}) \vee \text{happy}(X)$
2. $\neg \text{study}(Y) \vee \text{pass}(Y, Z)$
3. $\neg \text{lucky}(W) \vee \text{pass}(W, V)$
4. $\neg \text{study}(\text{john})$
5. $\text{lucky}(\text{john})$
6. $\neg \text{lucky}(U) \vee \text{win}(U, \text{lottery})$

Into these clauses is entered, in clause form, the negation of the conclusion:

7. $\neg \text{happy}(\text{john})$



Resolution refutation for the "happy student" problem.

4. All people that are not poor and are smart are happy. Those people that read are not stupid. John can read and is wealthy. Happy people have exciting lives. Can anyone be found with an exciting life?

We translate the story into the predicate calculus expressions:

$\forall X\ \neg poor(X) \wedge smart(X) \longrightarrow happy(X)$

$\forall Y\ read(Y) \longrightarrow smart(Y)$

(It would be much better to introduce $\forall Y\ read(Y) \longrightarrow \neg stupid(Y)$ together with additional axiom $\forall x\ (smart(x) \longrightarrow \neg stupid(x)) \wedge (smart(x) \longrightarrow \neg stupid(x))$, but I did not do that.)

$read(john) \wedge wealthy\ (john)$

$\forall Z\ happy(Z) \longrightarrow exciting(Z))$

The negation of the conclusion is: $\neg\exists W\ exciting(W)$

These predicate calculus expressions are transformed into the following clauses:

1. $poor(X) \vee \neg smart(X) \vee happy(X)$
2. $\neg read(Y) \vee smart(Y)$
3. $read(john)$
4. $\neg poor(john)$
5. $\neg happy(Z) \vee exciting(Z)$
6. $\neg exciting(W)$

Resolution proof for the "exciting life" problem