A Predicate Logic Example

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1. Marcus was a man.

Man(Marcus)

2. Marcus was a Pompeian.

Pompeian(Marcus)

3. All Pompeian were Romans.

$\forall x: Pomeian(x) \rightarrow Roman(x)$

4. Caesar was a Ruler.

Ruler(Caesar)

5. All Romans were either loyal to Caesar or hated him

$\forall x: Roman(x) \rightarrow loyalto(x, Caesar) \vee hate(x, Caesar)$

6. Everyone is loyal to someone.

$\forall x: \exists y: loyalto(x, y)$

7. Persons only try to assassinate rulers they are not loyal to.

$\forall x: \forall y: person(x) \wedge ruler(y) \wedge tryassassinate(x, y)$
$\rightarrow \neg loyalto(x, y)$

8. Marcus tried to assassinate Caesar.

tryassassinate(Marcus, Caesar)

9. All men are persons.

$\forall x: man(x) \rightarrow person(x)$

Overriding Defaults
Exception

Suppose we add:

a) $Pompeian(Paulus)$

b) $\neg$ [$loyalto(Paulus, Caesar) \vee hate(Paulus, Caesar)$]

But now we have a problem with 5:

$\forall x: Roman(x) \rightarrow loyalto(x, Caesar) \vee hate(x, Caesar)$

inconsistency

So, we have to change it to:

$\forall x: Roman(x) \wedge \neg eq(x, Paulus) \rightarrow$
$loyalto(x, Caesar) \vee hate(x, Caesar)$

Every exception to a general rule must be stated twice.

gt(1, 0) ***ln(0, 1)***
gt(2, 1) ***ln(1, 2)***
gt(3, 2) ***ln(2, 3)***

. . . . . .

Computable Predicates and Functions

gt(2+3, 1)
5 > 1

Another Predicate Logic Example

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1. Marcus was a man.
2. Marcus was a Pompeian.
3. Marcus was born in 40 A.D.
4. All men are mortal.
5. All Pompeians died when the volcano erupted in 79 A.D.
6. No mortal lives longer than 150 years.
7. It is now 2010.

Is Marcus alive?

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8. Alive means not dead.
9. If someone dies, then he is dead at all later times.

A Set of Facts about Marcus

1. $man(Marcus)$
2. $Pompeian(Marcus)$
3. $born(Marcus, 40)$
4. $\forall x: man(x) \rightarrow mortal(x)$
5. $\forall x: Pompeian(x) \rightarrow died(x, 79)$
6. $erupted(volcano, 79)$
7. $\forall x: \forall t_1: \forall t_2: mortal(x) \wedge born(x, t_1) \wedge gt(t_2 - t_1, 150) \rightarrow dead(x, t_2)$
8. $now = 2010$

9. $\forall x: \forall t: \underline{(alive(x,t) \rightarrow \neg dead(x, t))} \wedge (\neg dead(x,t) \rightarrow alive(x,t))$
10. $\forall x: \forall t_1: \forall t_2: died(x, t_1) \wedge gt(t_2, t_1) \rightarrow dead(x, t_2)$

$(A \rightarrow \neg D) \wedge (\neg D \rightarrow A)$

P	Q	$\neg P$	$\neg P \vee Q$	$P \rightarrow Q$	$(\neg P \vee Q) = (P \rightarrow Q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Truth table demonstrating the equivalence of $P \rightarrow Q$ and $\neg P \vee Q$.

But we know that $X \rightarrow Y = \neg X \vee Y$, then

$(A \rightarrow \neg D) = (\neg A \vee \neg D) = (\neg D \vee \neg A) = D \rightarrow \neg A$, consequently,

$\forall x: \forall t: (dead(x,t) \rightarrow \neg alive(x,t))$

One Way of Proving That Marcus Is Dead

$\neg$*alive(Marcus, now)*

↑(9, substitution)

dead(Marcus, now)

↑(10, substitution)

died(Marcus, t_1) $\wedge$ *gt(now, 79)*

↑(5, substitution)

Pompeian(Marcus) $\wedge$ *gt(now, 79)*

↑(2)

gt(now, 79)

↑(8, substitute equals)

gt(2010, 79)

↑(compute *gt*)

nil

Another Way of Proving That Marcus Is Dead

$\neg alive(Marcus, now)$

↑(9, substitution)

$dead(Marcus, now)$

↑(7, substitution)

$mortal(Marcus) \wedge born(Marcus, t_1) \wedge gt(now - t_1, 150)$

↑(4, substitution)

$man(Marcus) \wedge born(Marcus, t_1) \wedge gt(now - t_1, 150)$

↑(1)

$born(Marcus, t_1) \wedge gt(now - t_1, 150)$

↑(3)

$gt(now - 40, 150)$

↑(8)

$gt(2010 - 40, 150)$

↑(compute minus)

$gt(1970, 150)$

↑(compute *gt*)

nil